Inspecting a set of strips optimally

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Braunschweig University of Technology and University of Bonn

Watchman and Touring problems

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- Path that visits or *sees* objects

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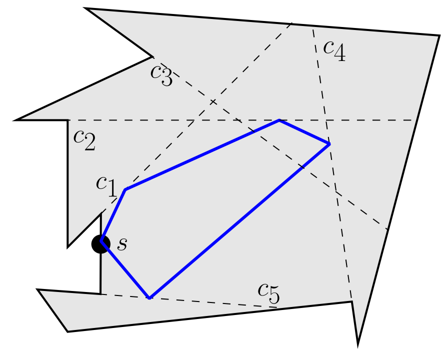

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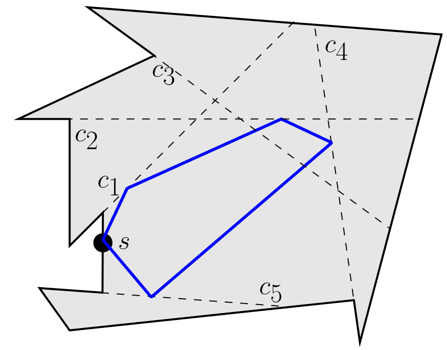

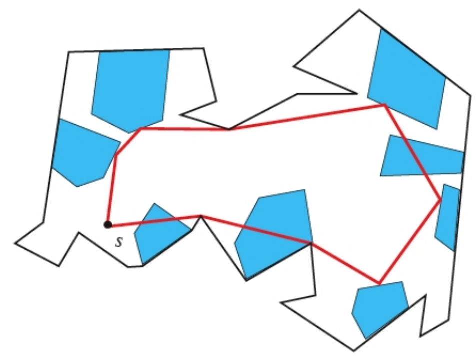

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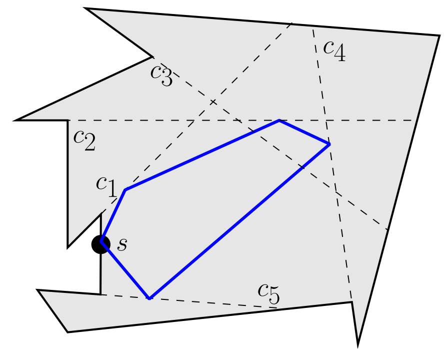

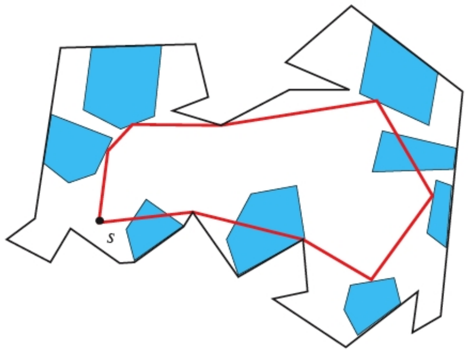

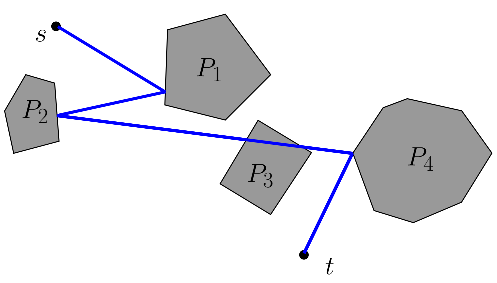

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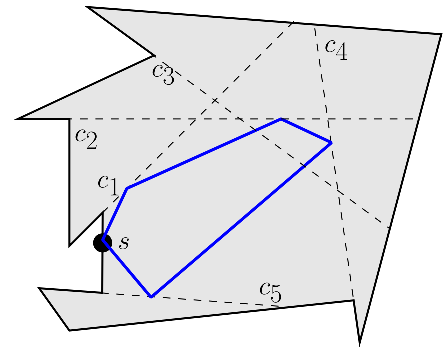

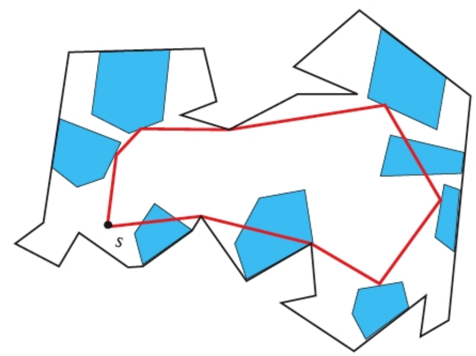

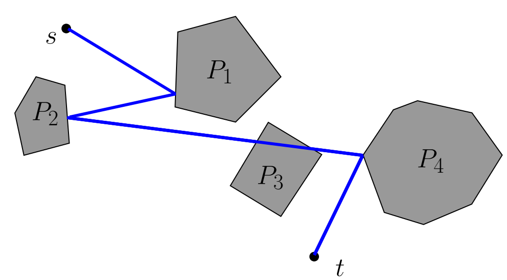

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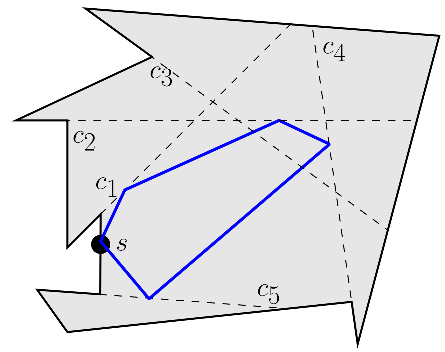

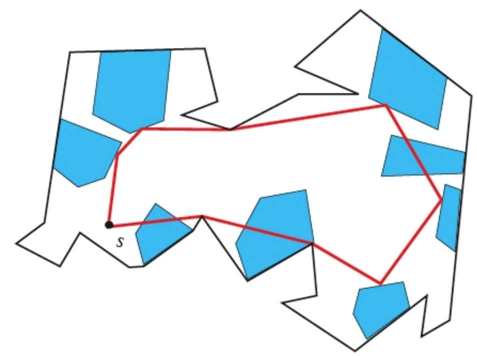

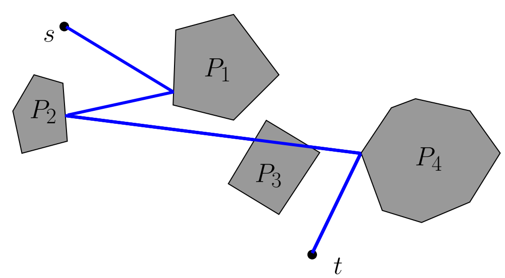

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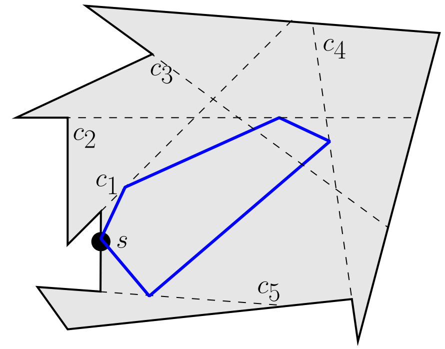
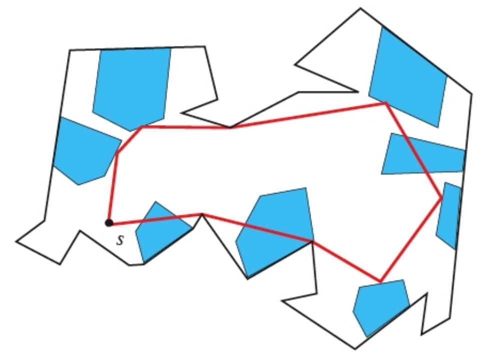
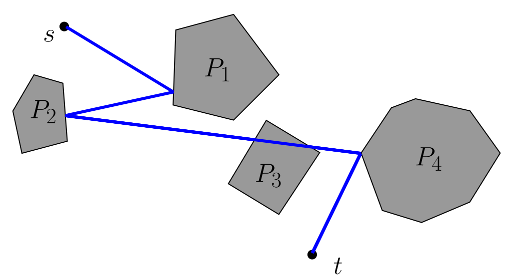
- Inspection path: time/path







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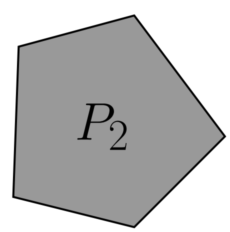

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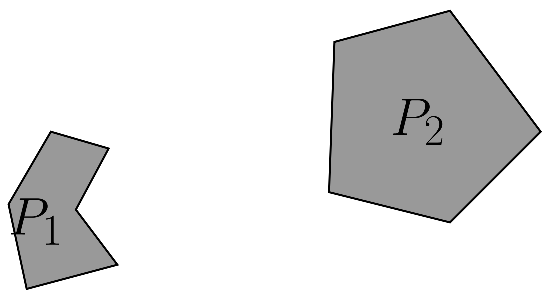

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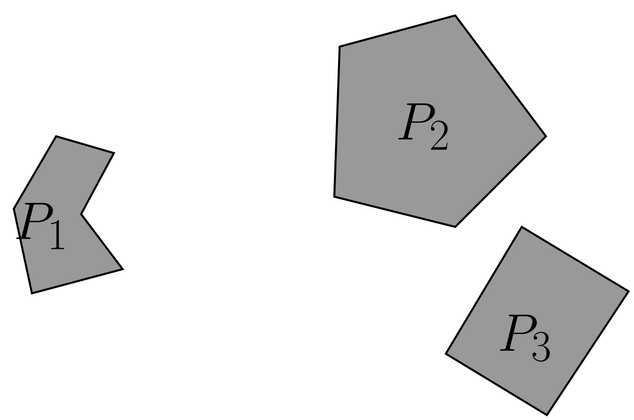

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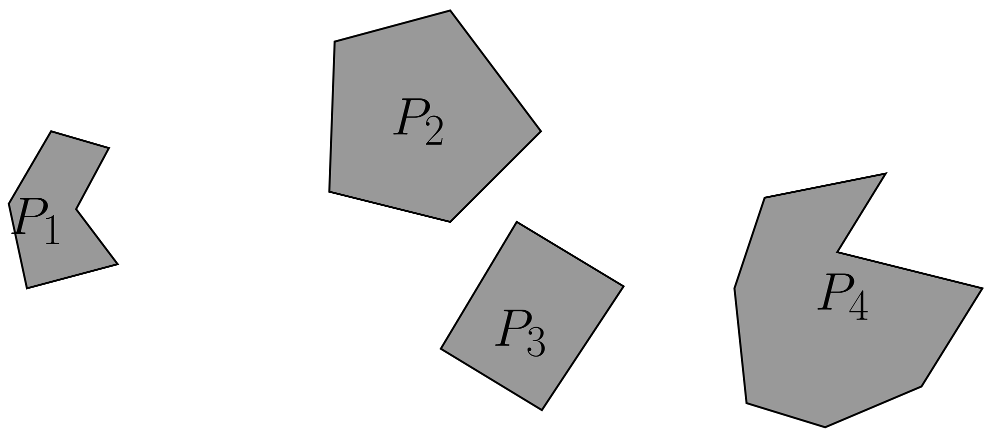

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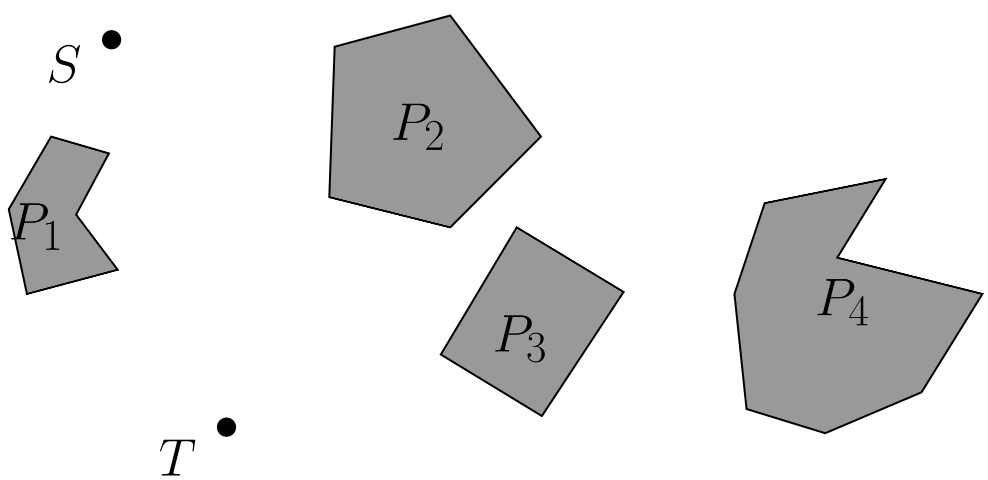

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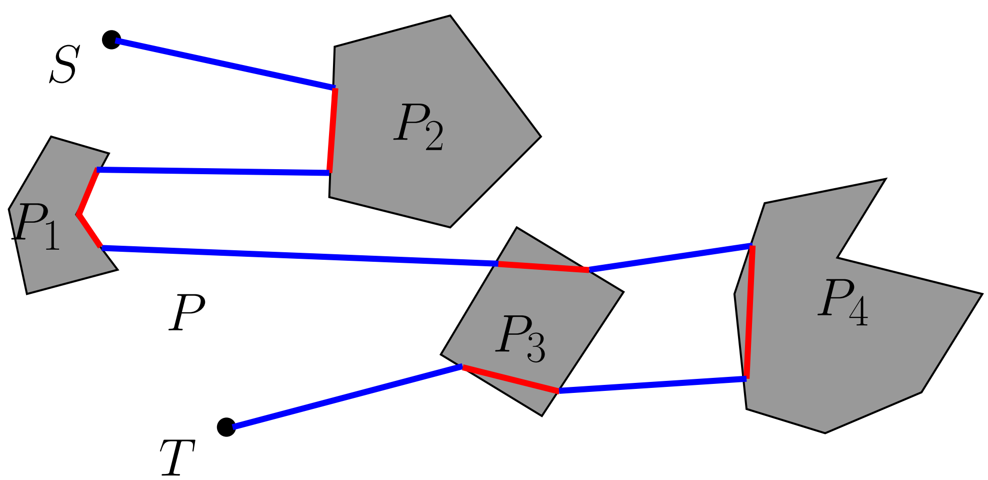

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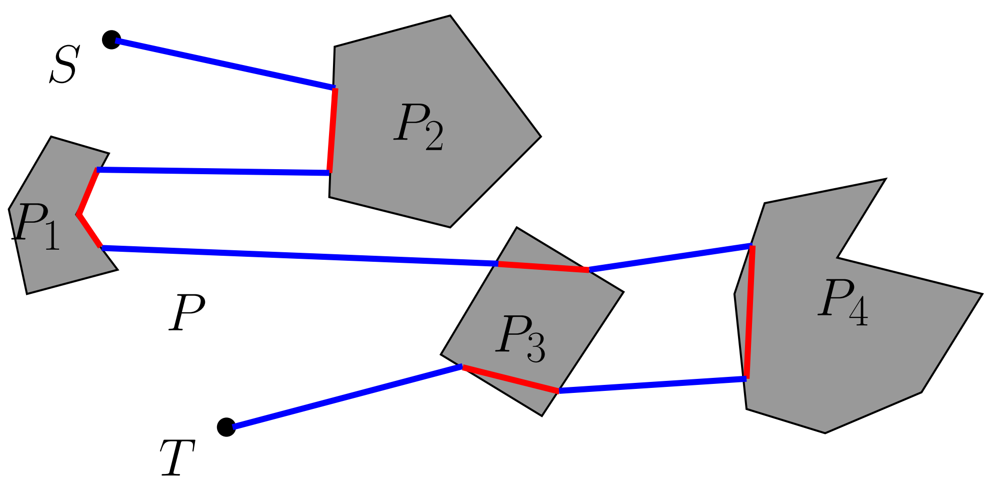

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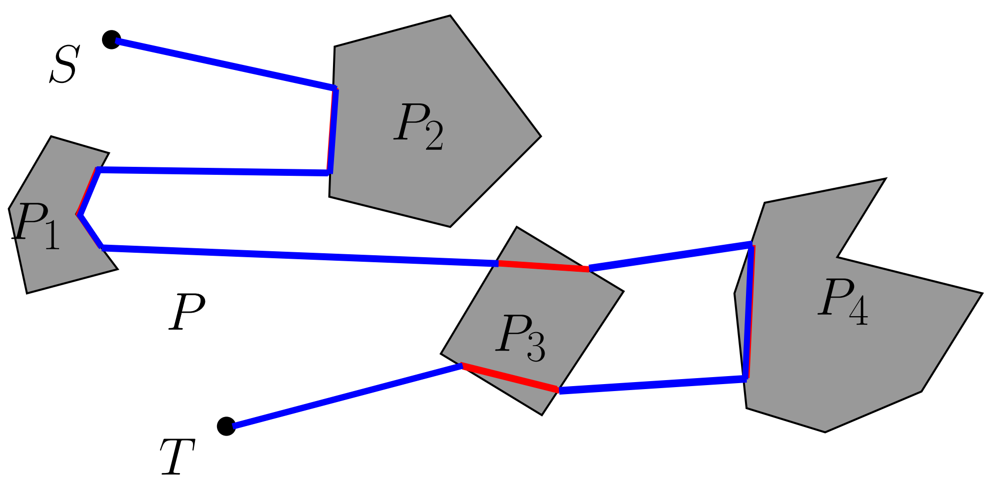

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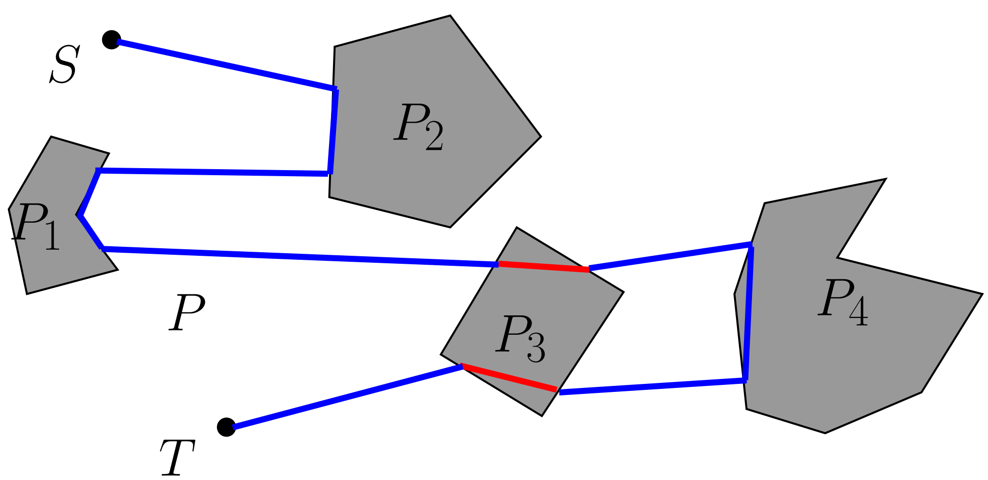

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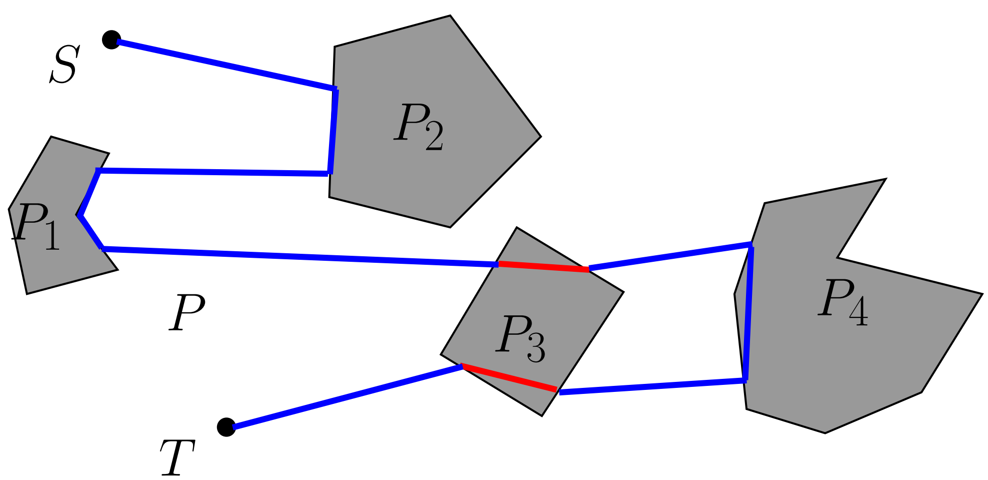

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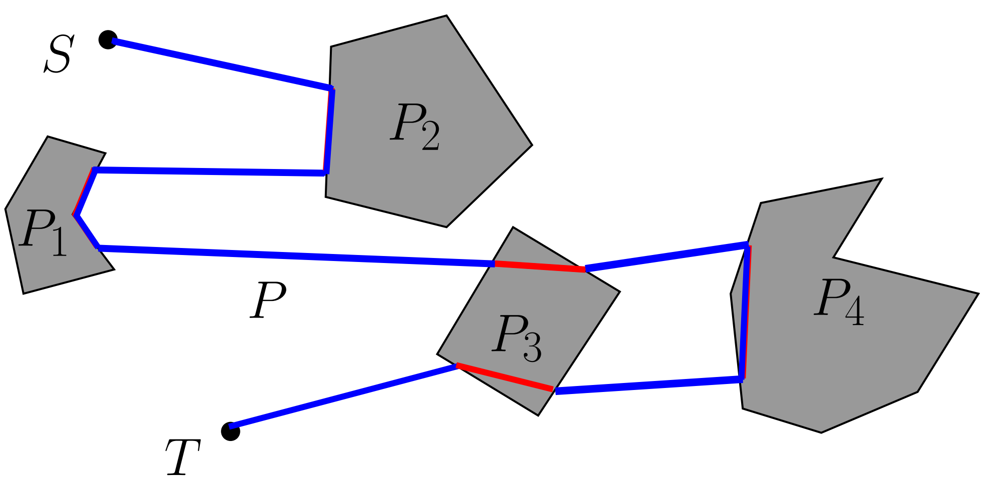

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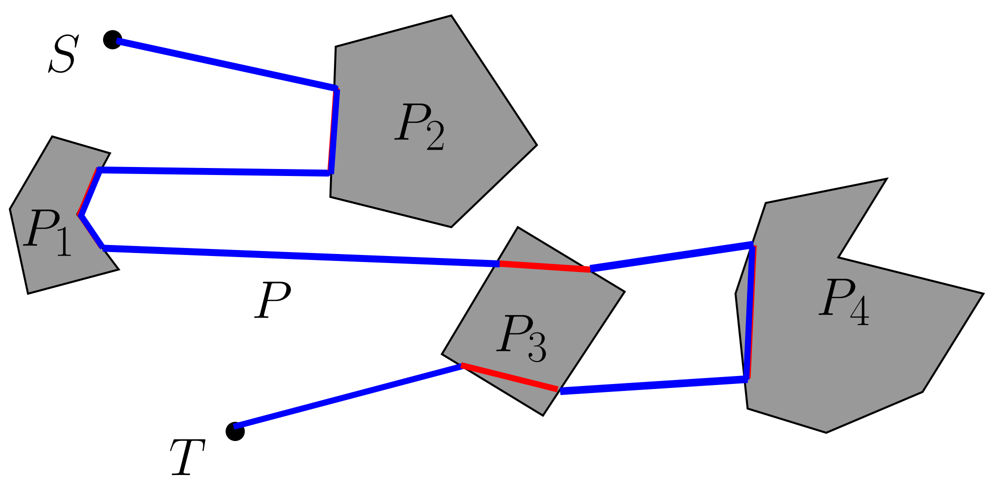

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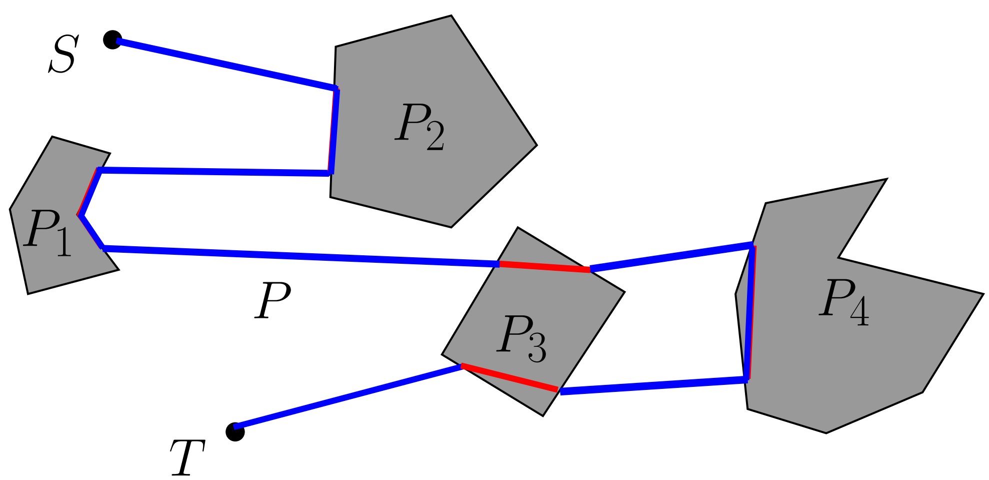

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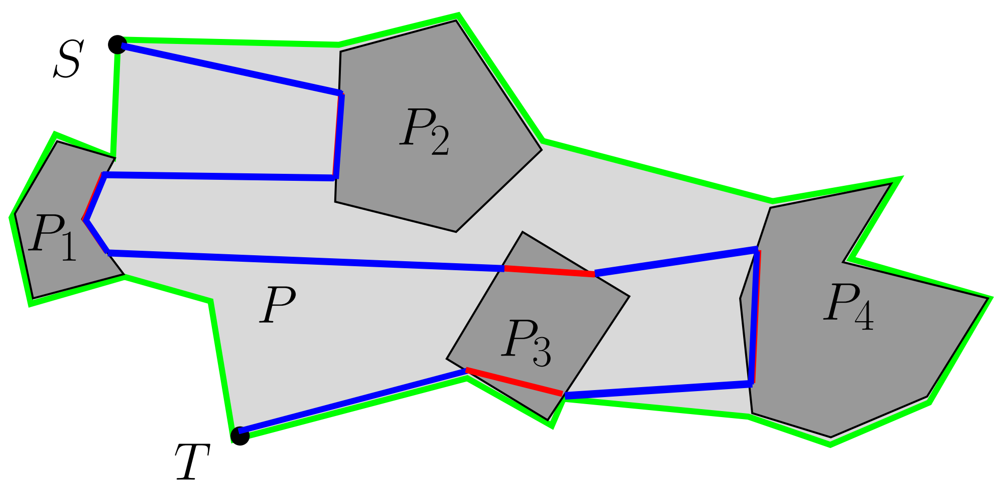

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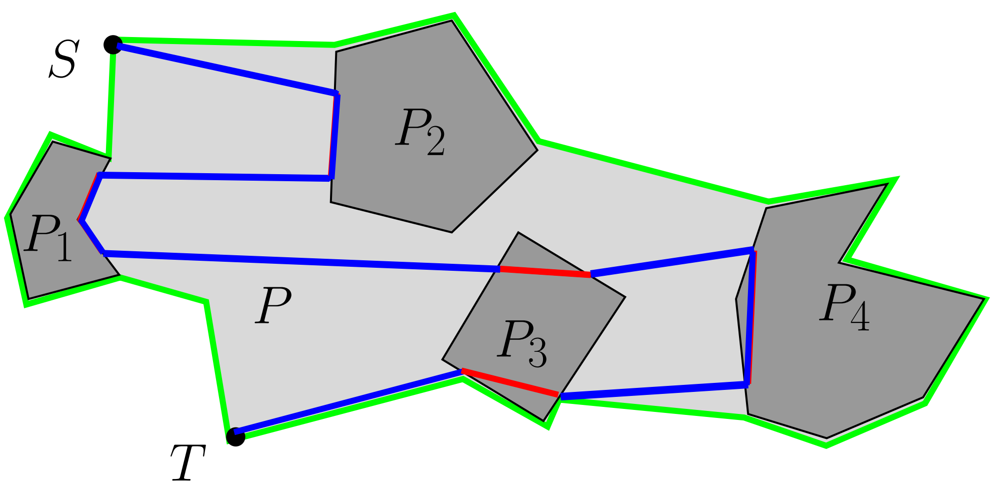

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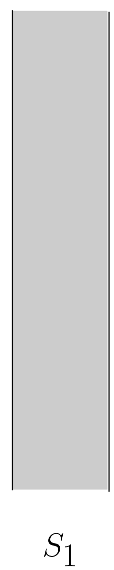
- Strips $S_1, S_2, \ldots, S_n$ of different widths $w_1, \ldots, w_n$



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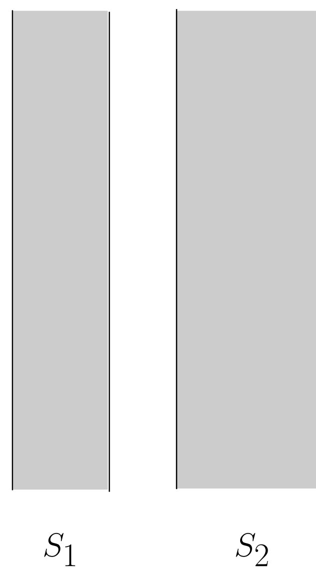

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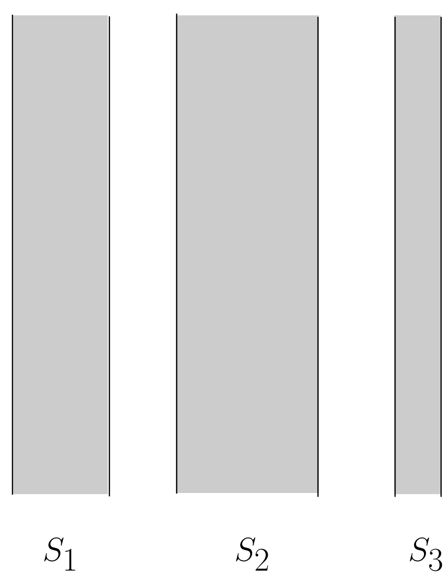

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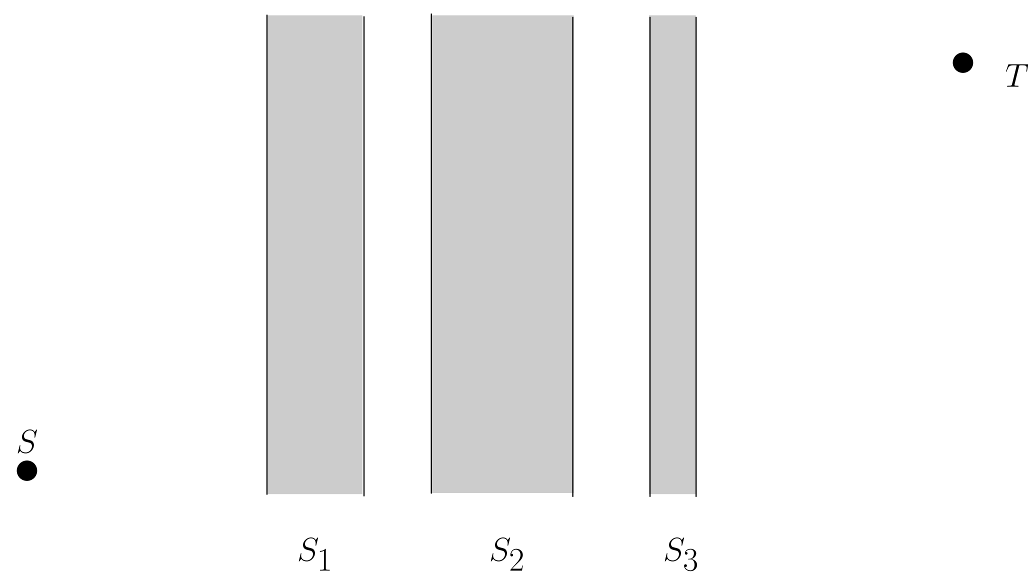

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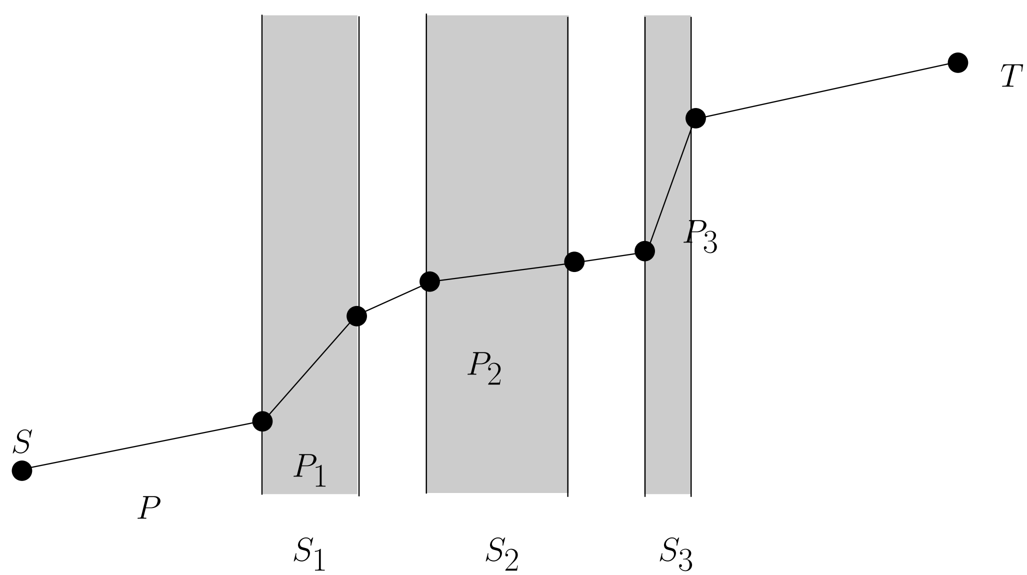

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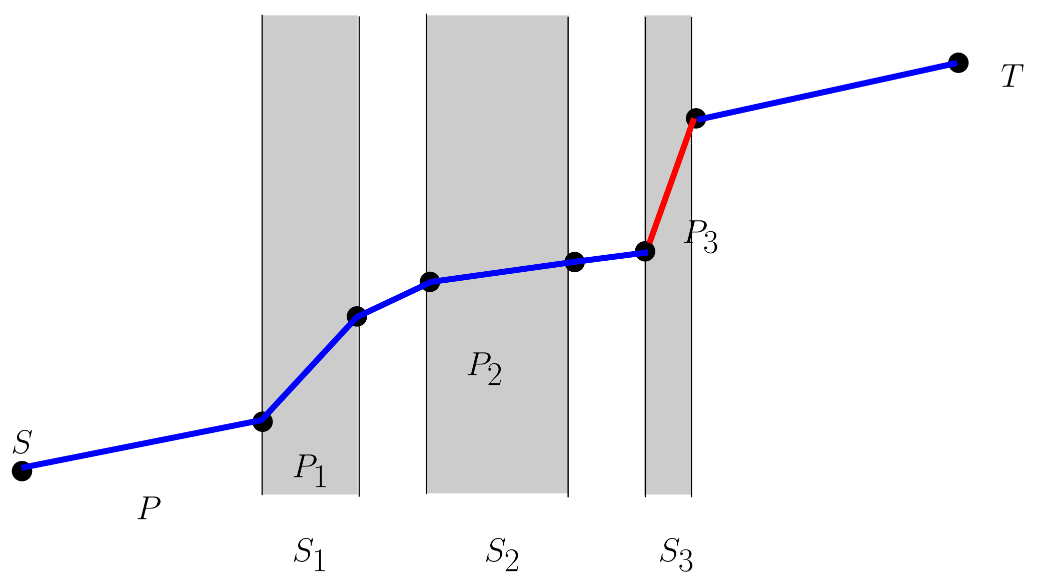

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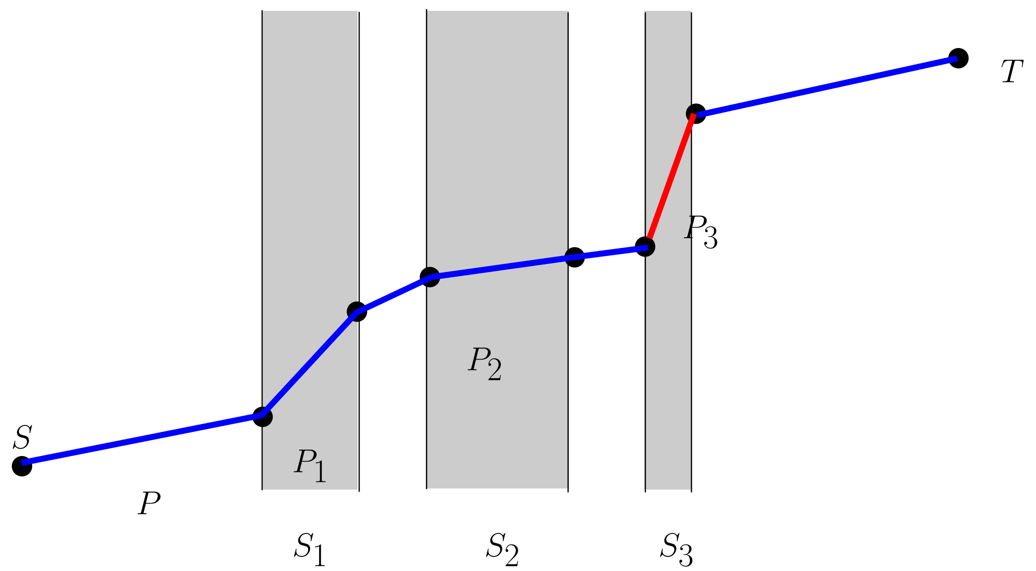

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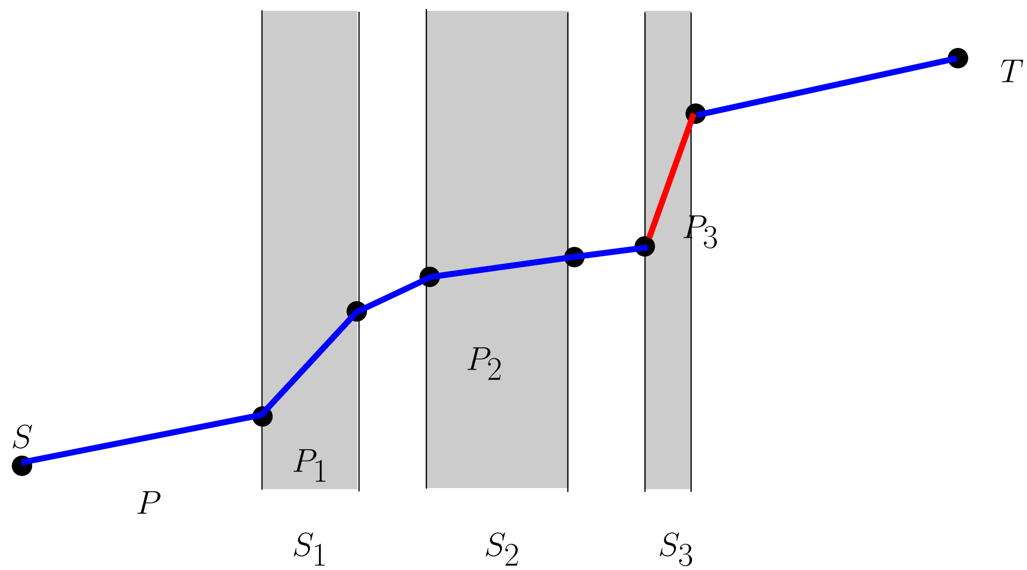

Inspection of a set of strips

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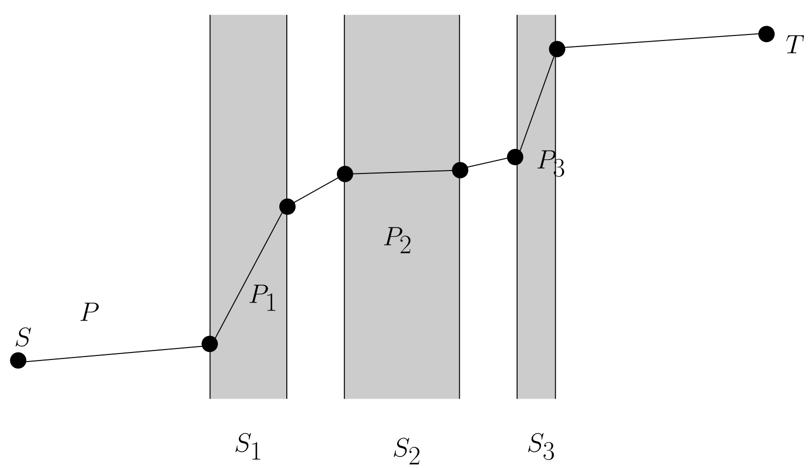
- Structural properties, reordering: Strips by increasing widths



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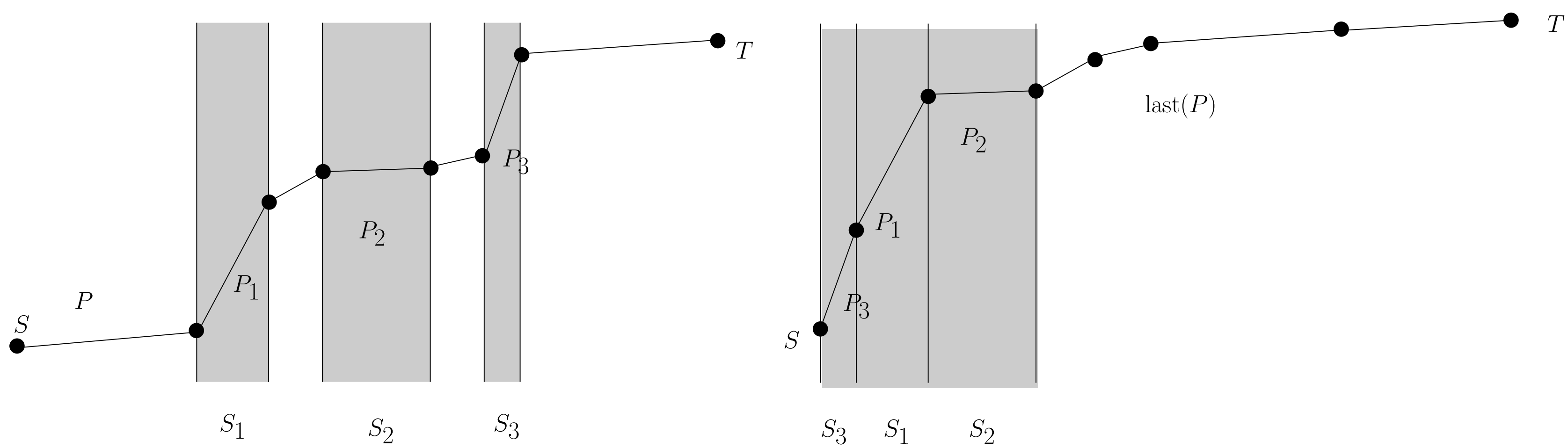

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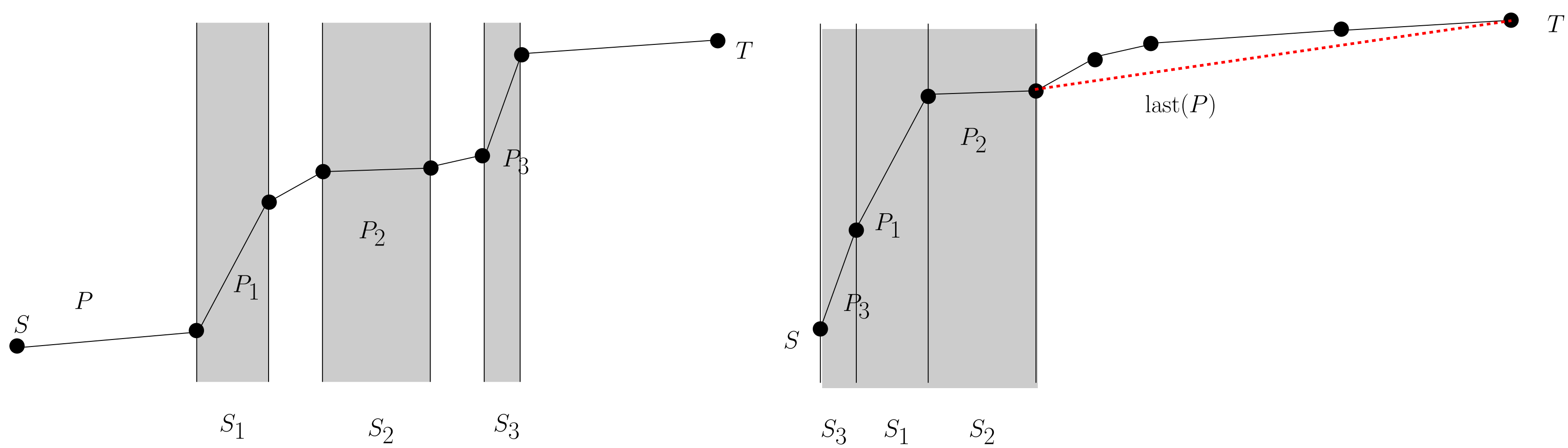

Inspection of a set of strips

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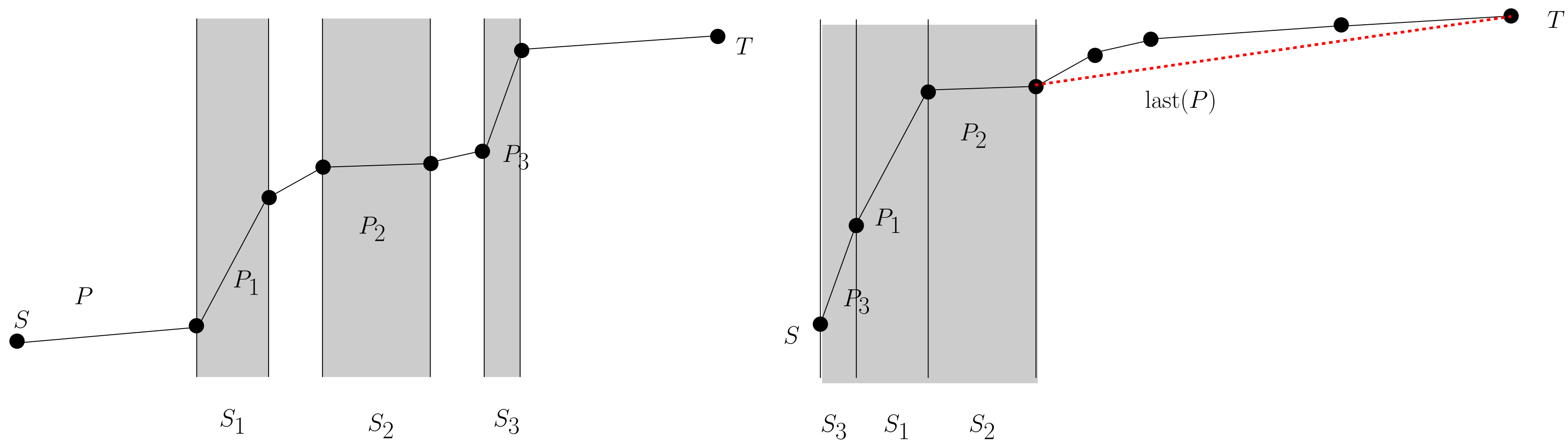
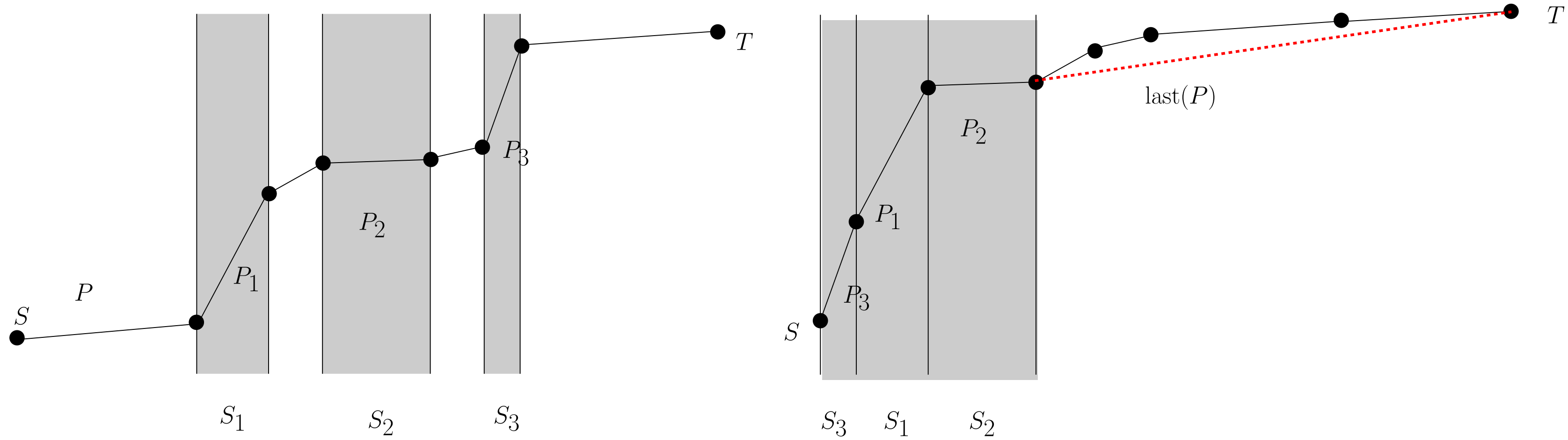
- Lemma: Kinks at the boundaries, the same slope between two successive strips

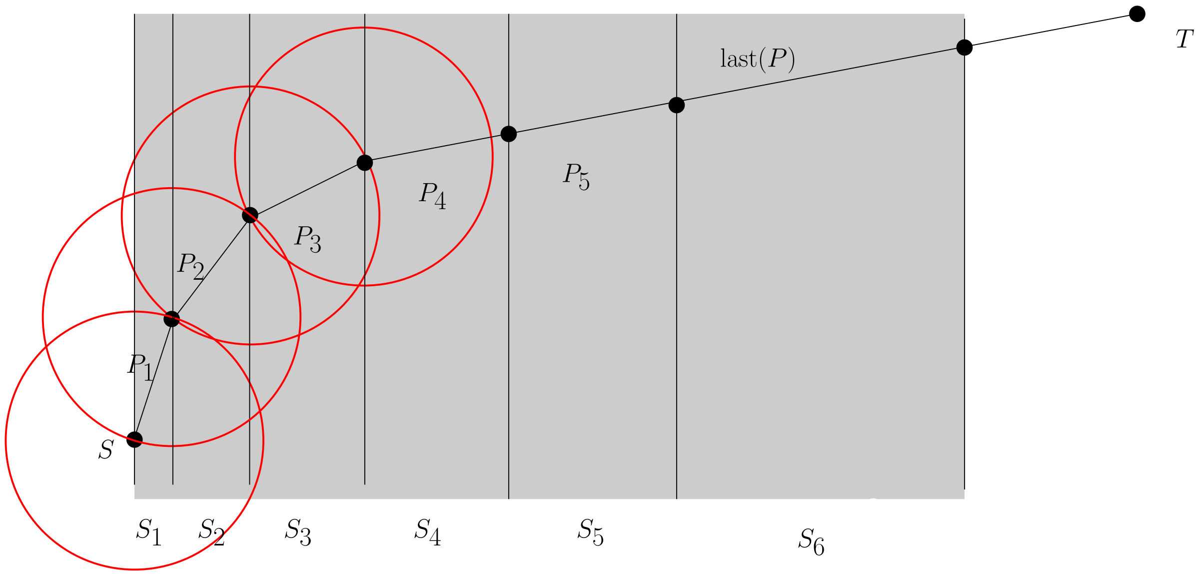


Inspection of a set of strips

- Structural properties, reordering: Strips by increasing widths
- Lemma: Kinks at the boundaries, the same slope between two successive strips
- In the following: n strips sorted by widths $w_i \leq w_{i+1}$ for $i = 1, \ldots, n-1$

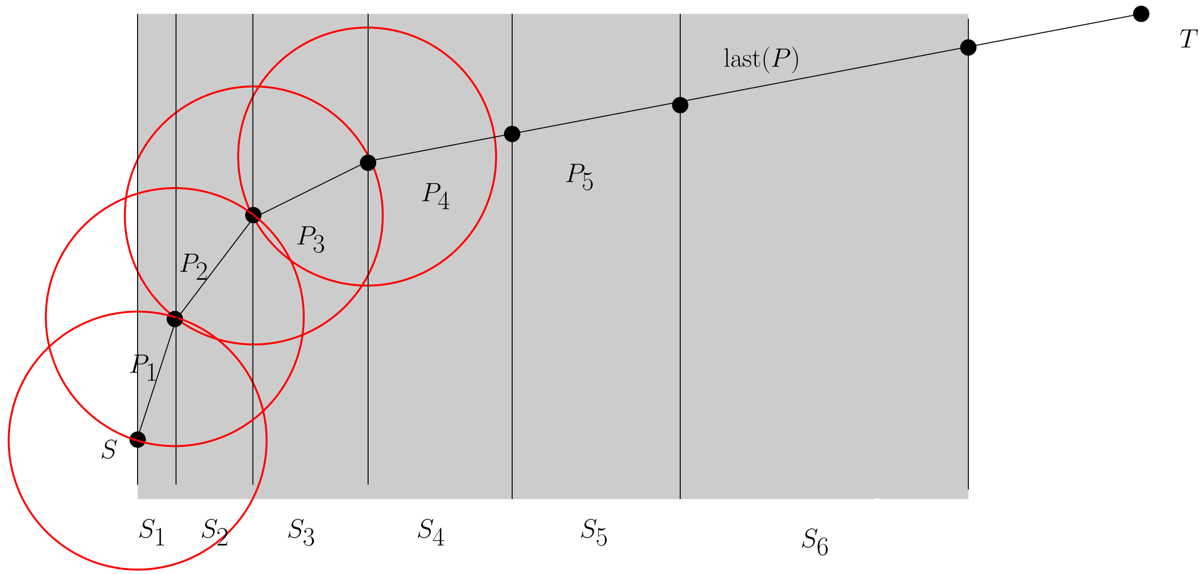


Structural properties: Relevant strips



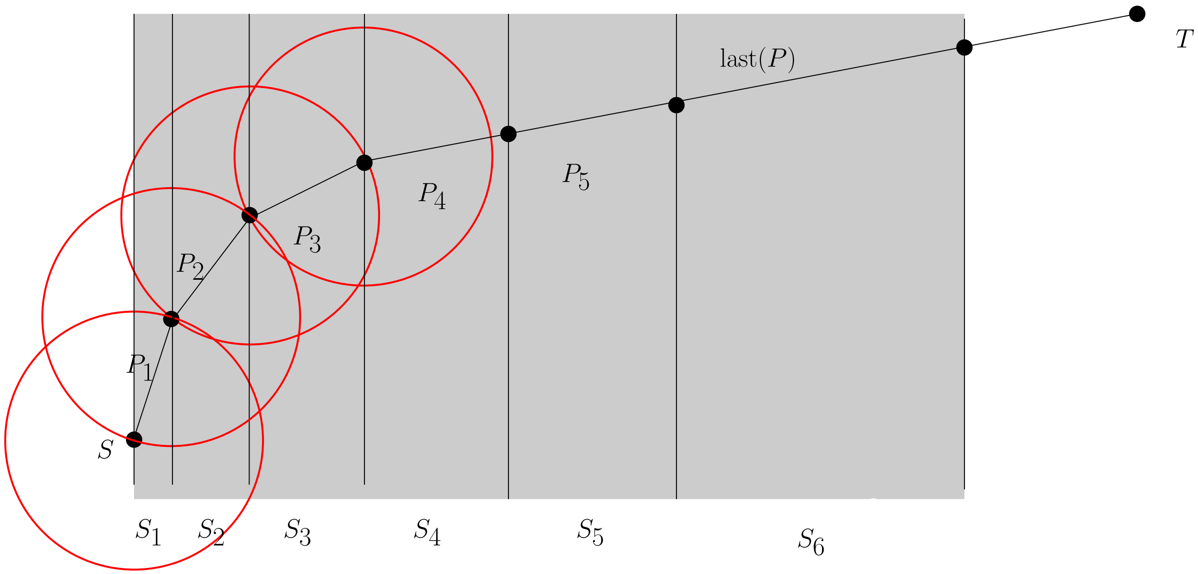
Structural properties: Relevant strips

- Sorted by widths, starting from the left, without gaps



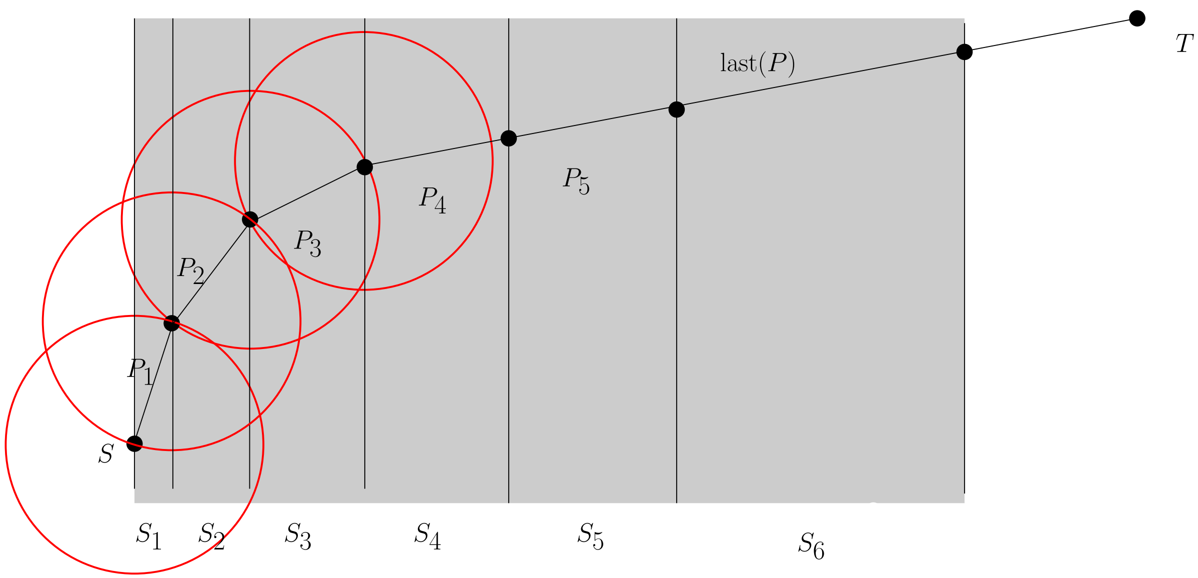
Structural properties: Relevant strips

- Sorted by widths, starting from the left, without gaps
- Visit the first k smallest strips with the same distance d



Structural properties: Relevant strips

- Sorted by widths, starting from the left, without gaps
- Visit the first k smallest strips with the same distance d
- Pass the rest with a distance $> d$



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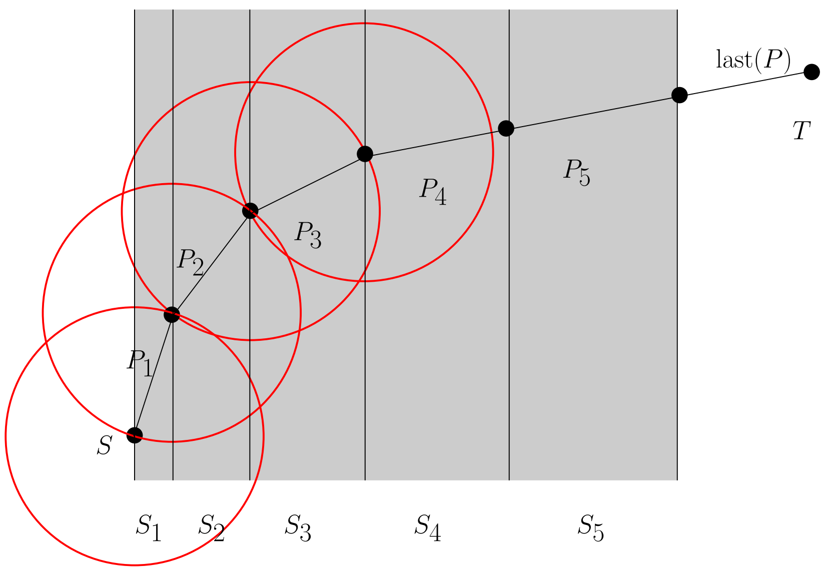
- Lemma: For optimal solution $\exists K \leq n$: $|P_i| = d$ for $i = 1, \ldots K$, $|P_i| > d$ for $i = K+1, \ldots n$, konvex wrt. ST



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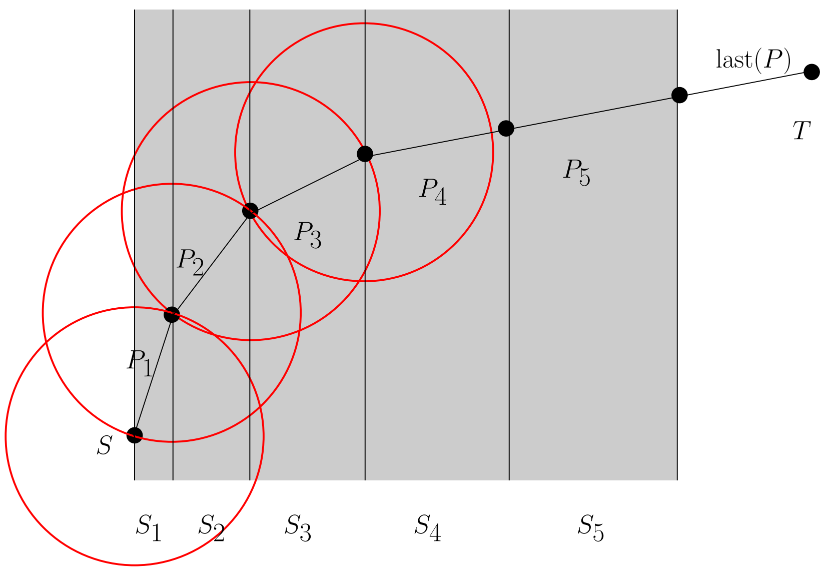

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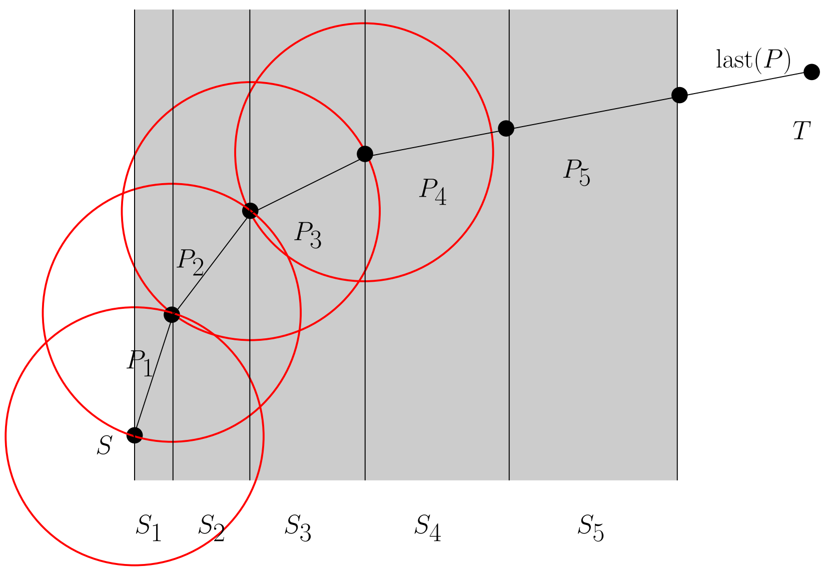

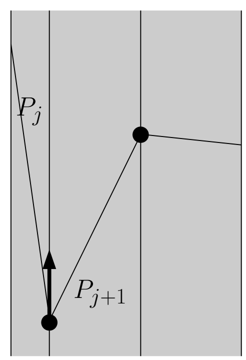

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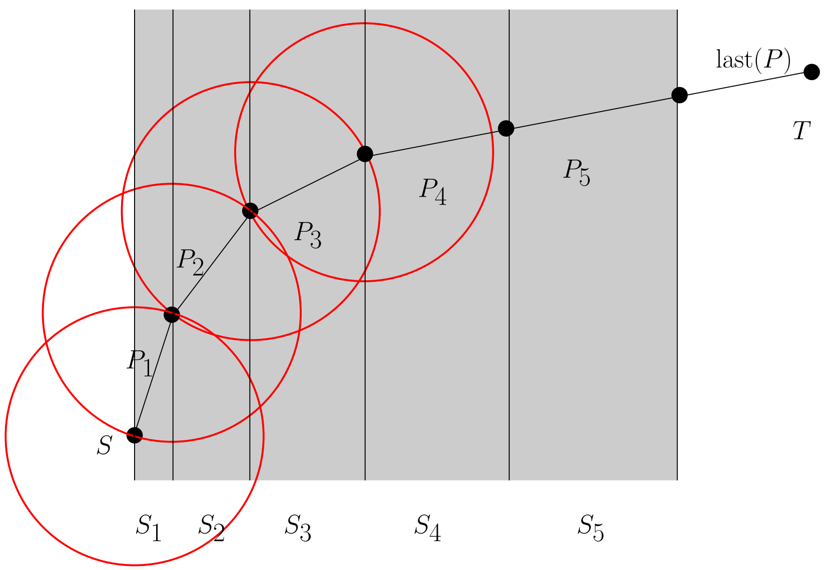
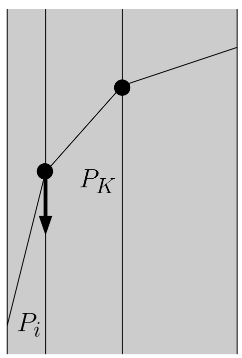
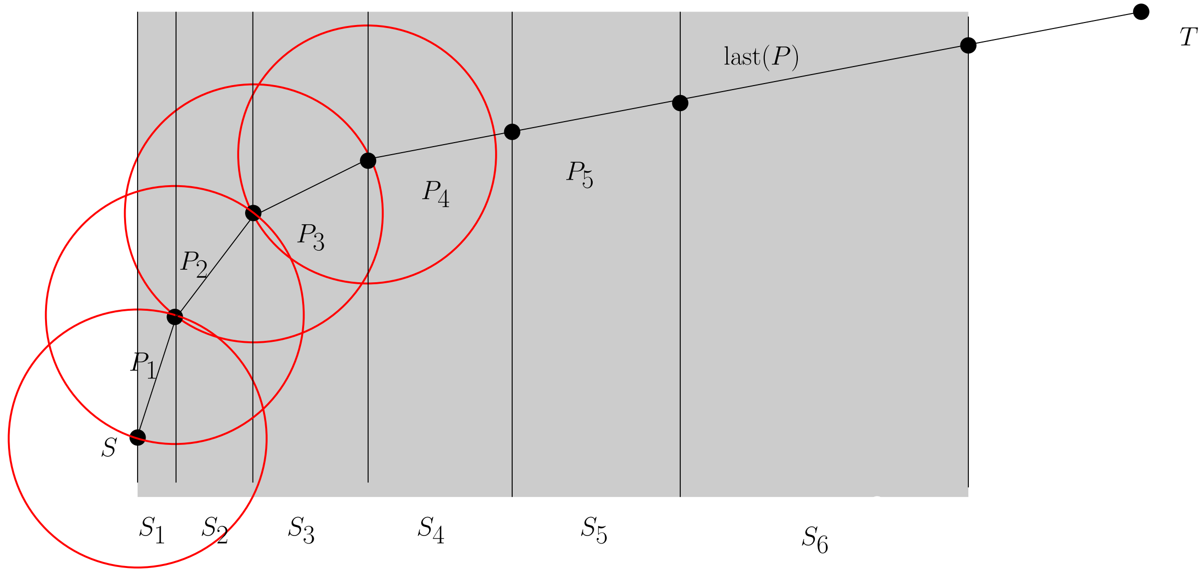
- A) Optimal path is monotonically increasing
- B) Ind. K s.th. P_K opt.: $|P_i| > |P_K|$, $w_i \leq w_K$ is impossible





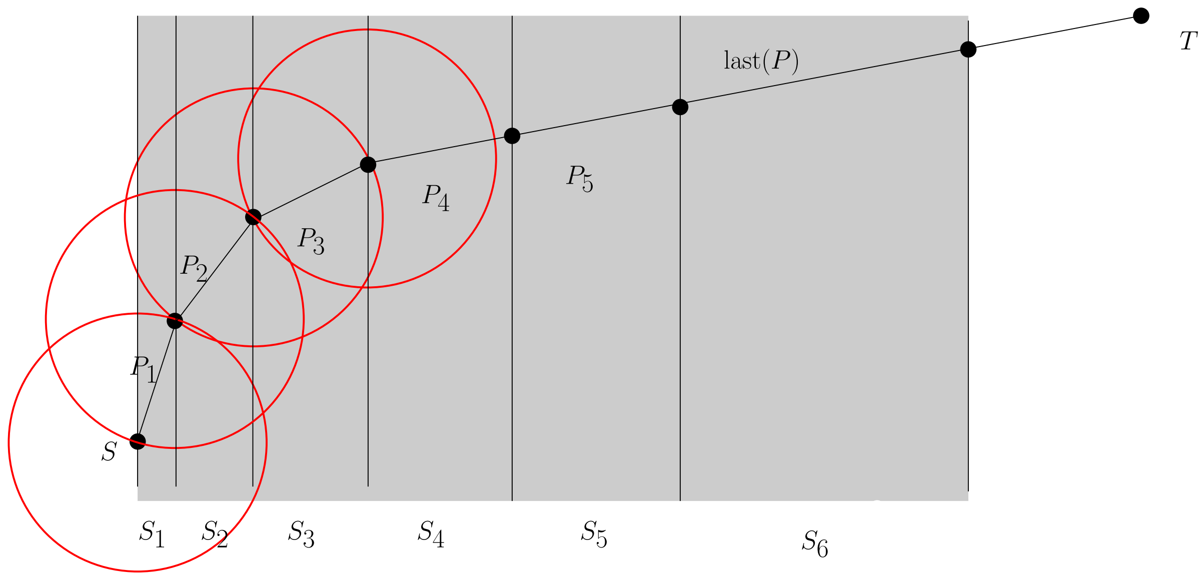
B)

Structural properties: Relevant strips



Structural properties: Relevant strips

- Incremental approach: Succ. add more strips sorted by widths



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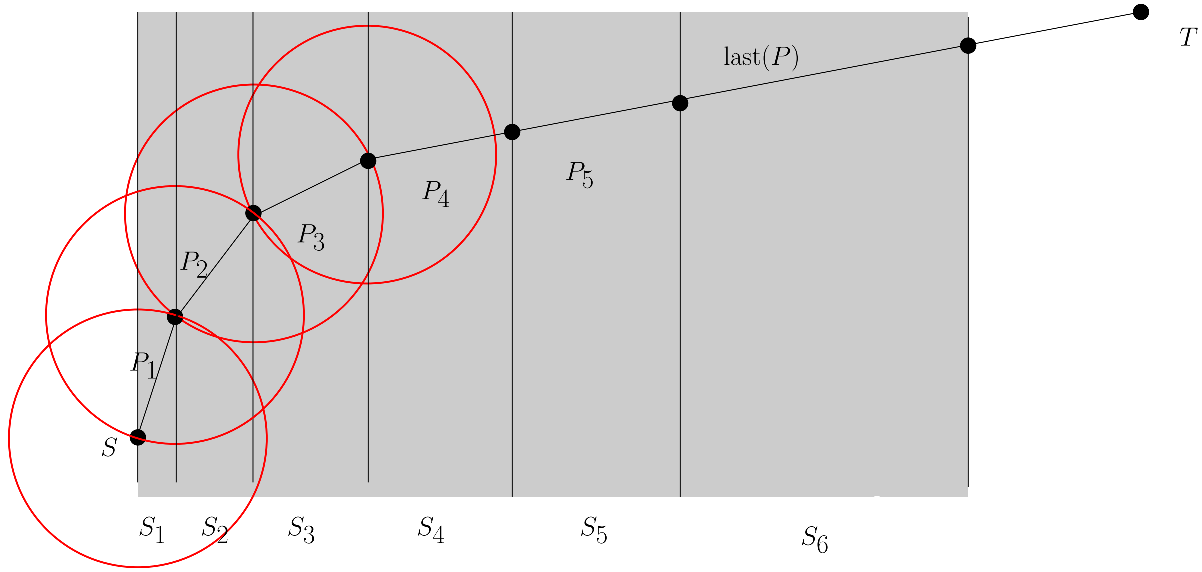

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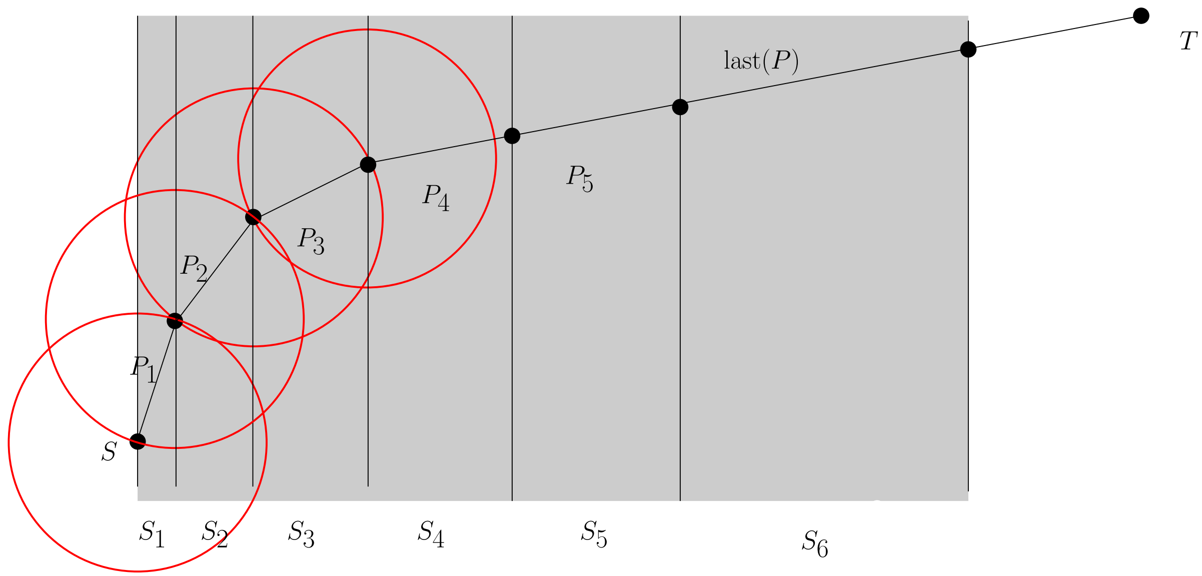

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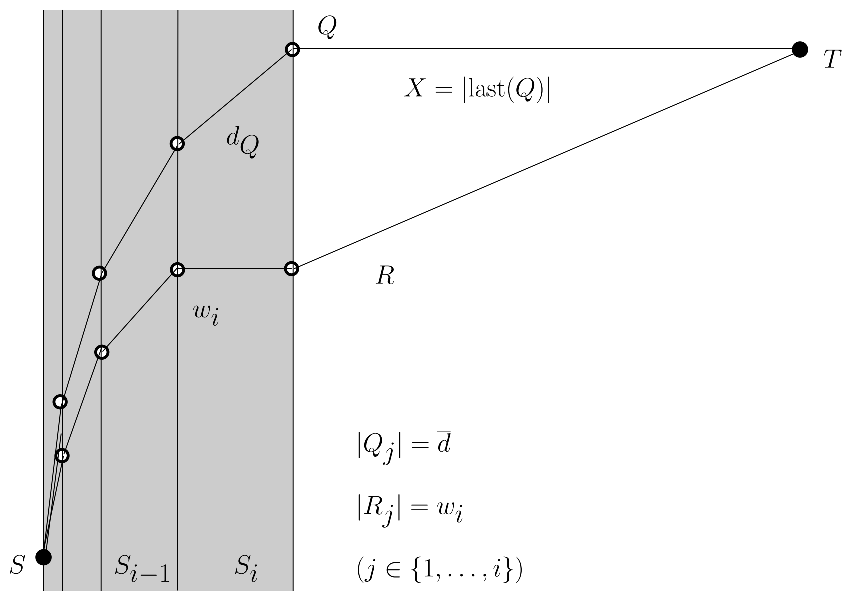

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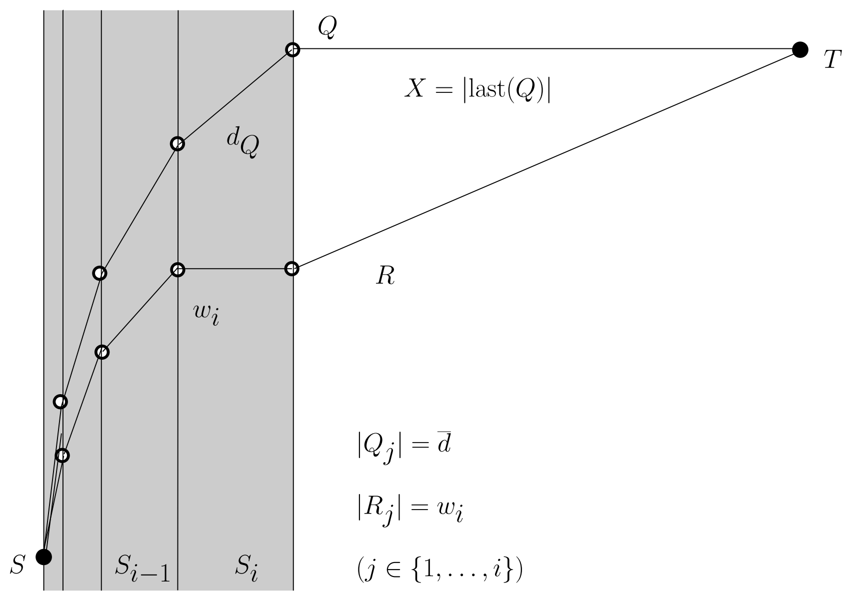

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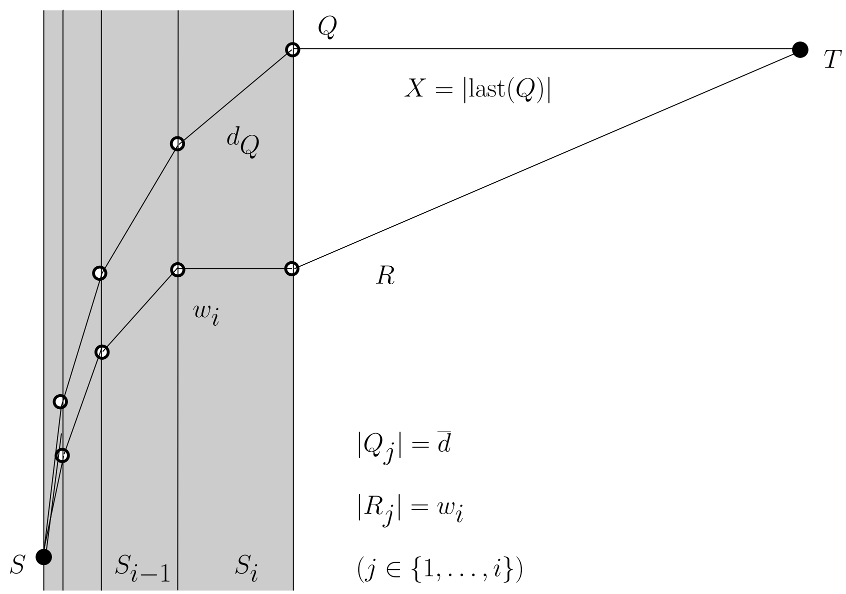

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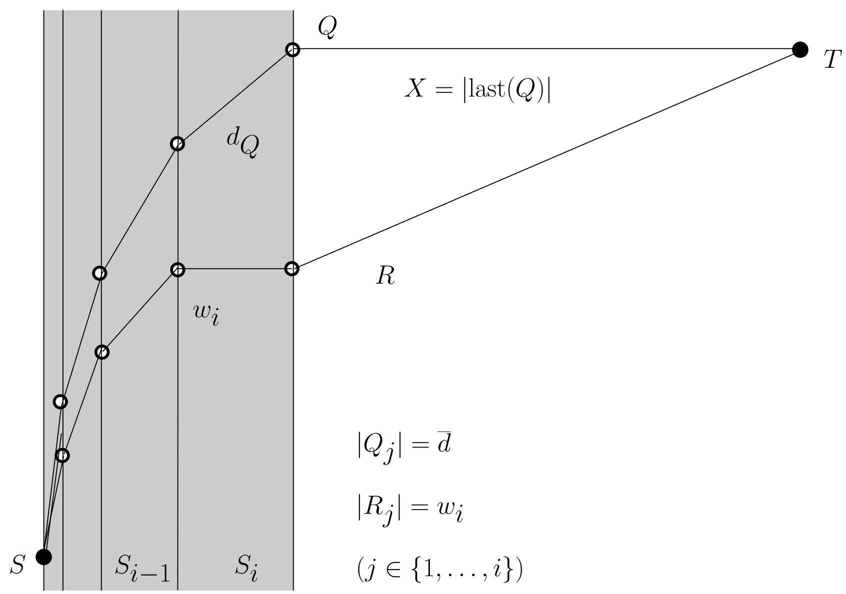
- Start with $d = w_i$, end with $d = d_Q$
- $\min_d f_i(d) := d \times (i-1) + \sqrt{X^2 + \left(t_y - \sum_{j=1}^{i} \sqrt{d^2 - w_j^2}\right)^2}$



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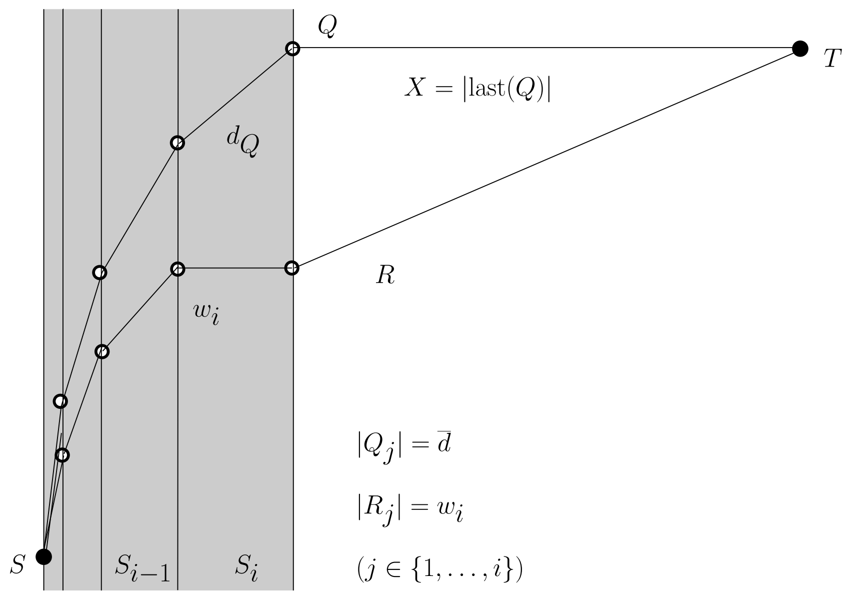
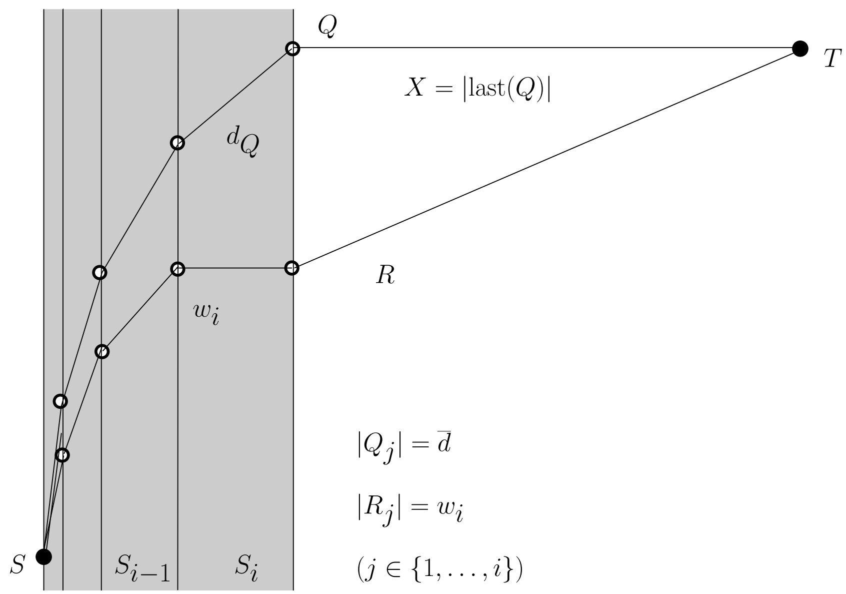


Optimal d_i with $|P_j| = d_i$ for $j = 1, \ldots i$

- Solve:

$$f_i'(d) = i - 1 - \frac{\left(t_y - \sum_{j=1}^{i} \sqrt{d^2 - w_j^2}\right)}{\sqrt{X^2 + \left(t_y - \sum_{j=1}^{i} \sqrt{d^2 - w_j^2}\right)^2}} \cdot \left(\sum_{j=1}^{i} \frac{d}{\sqrt{d^2 - w_j^2}} \right)$$



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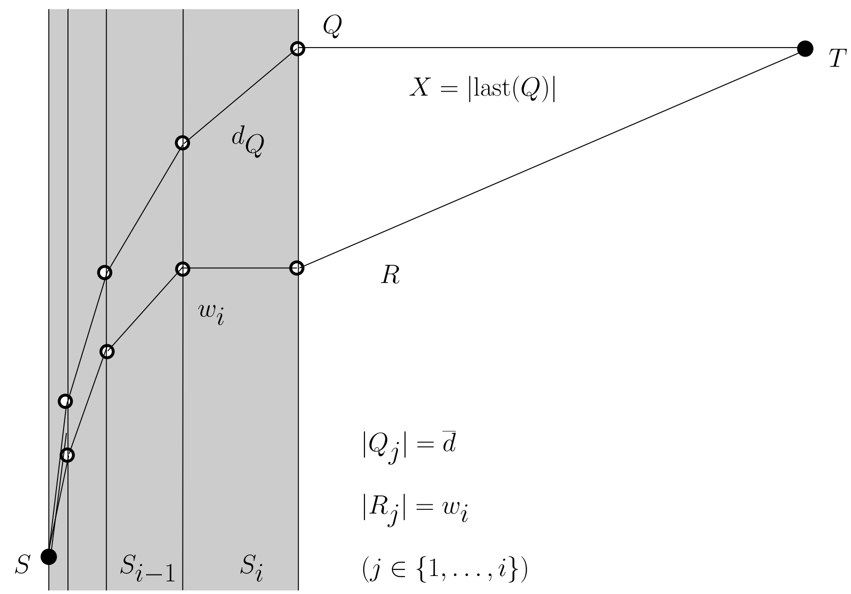
- Analysis: $f_i'(d)$ is monotone, one minimum at d_i for every i



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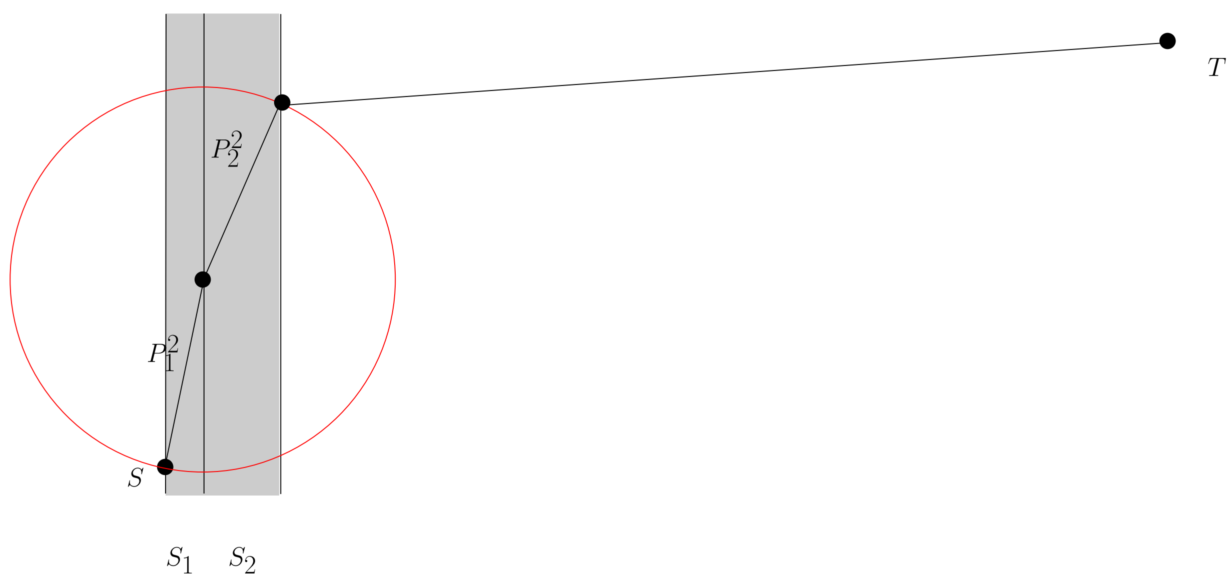

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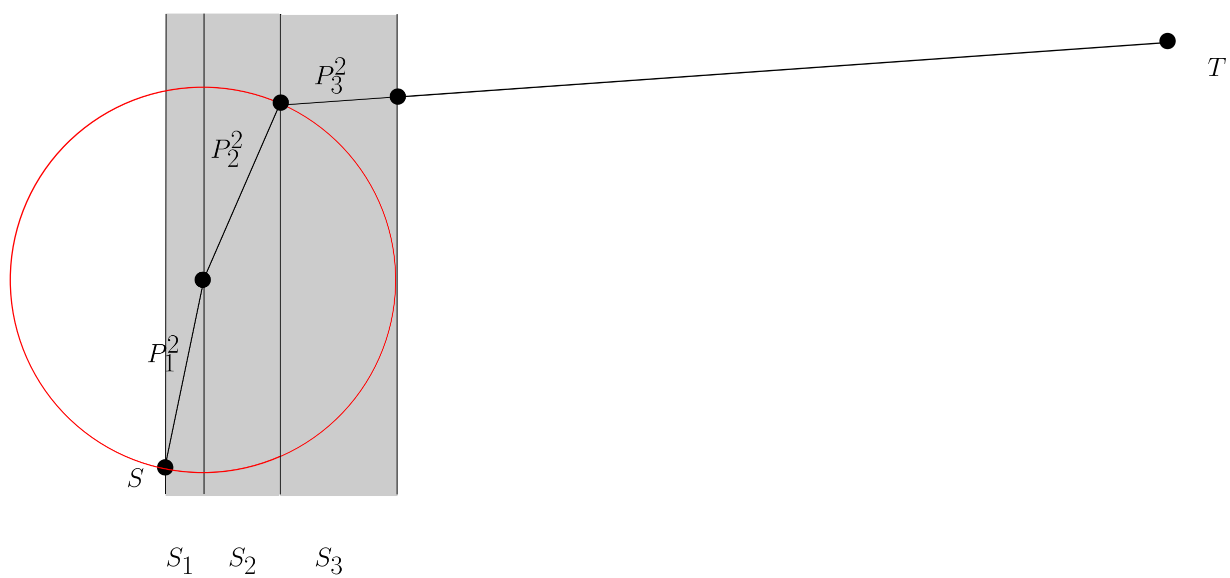

$d_2 = |P_2^2| = |P_1^2| \geq |P_3^2|$

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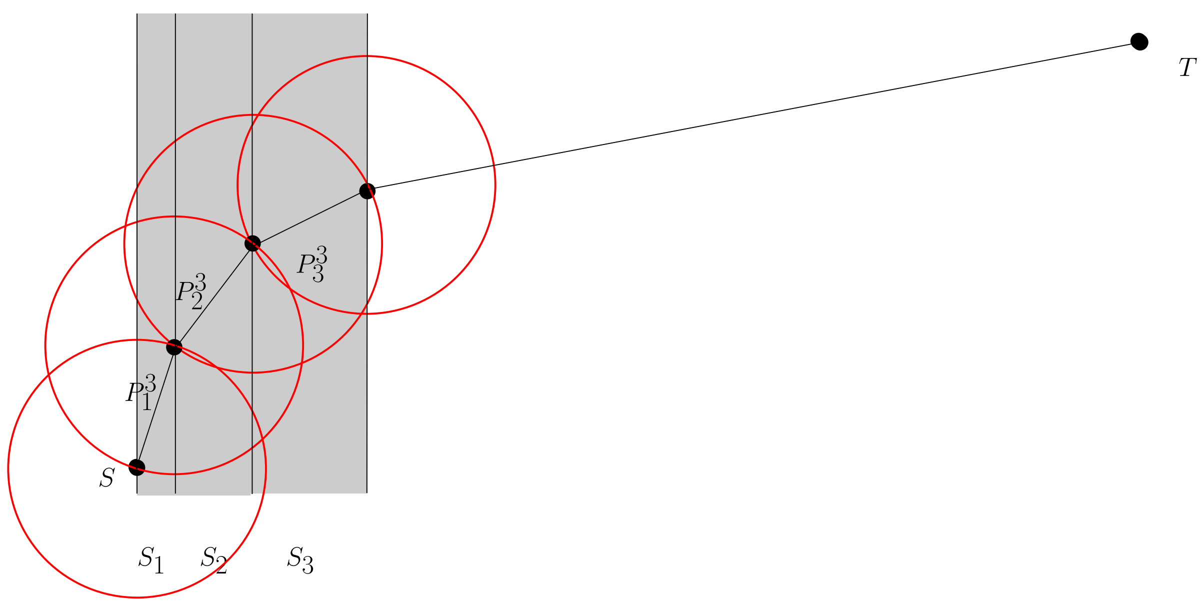

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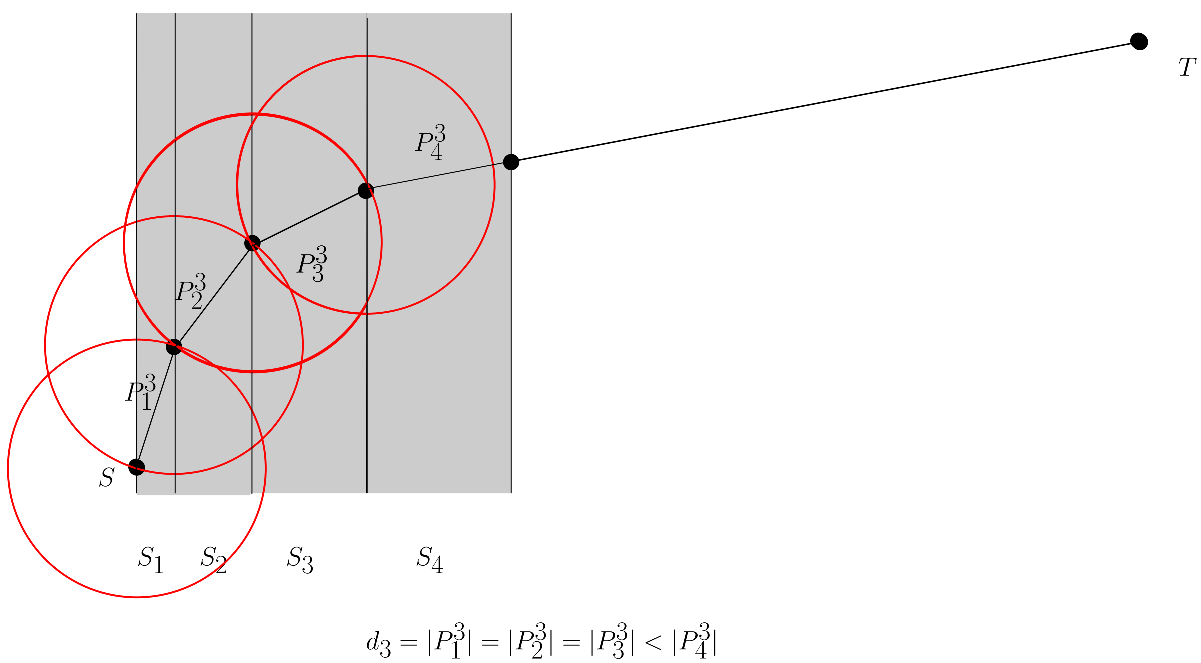

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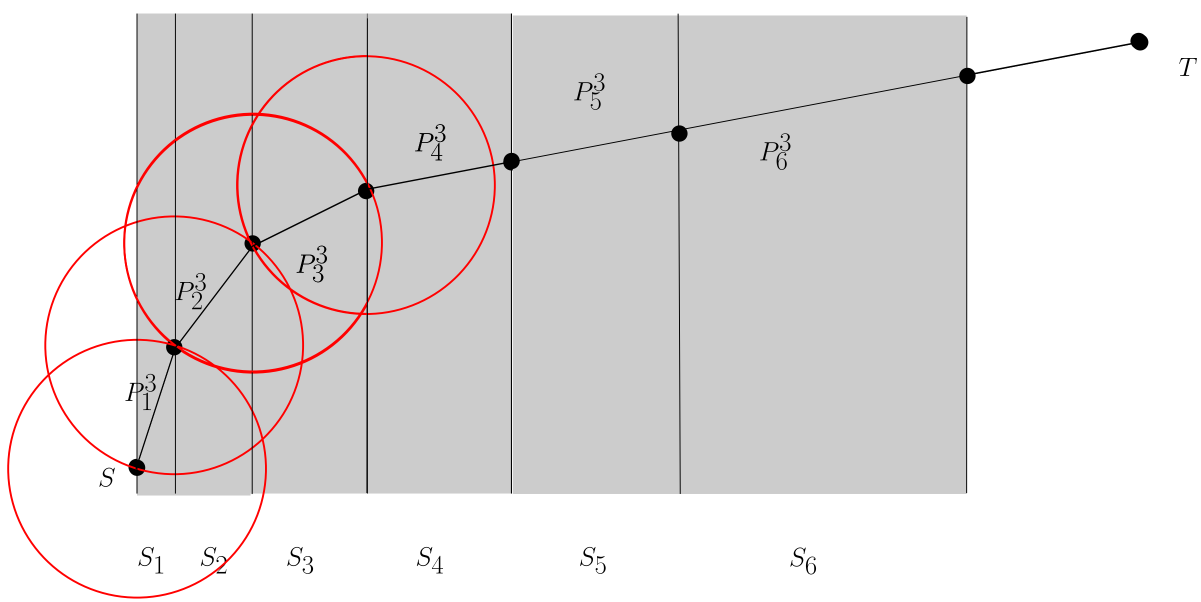

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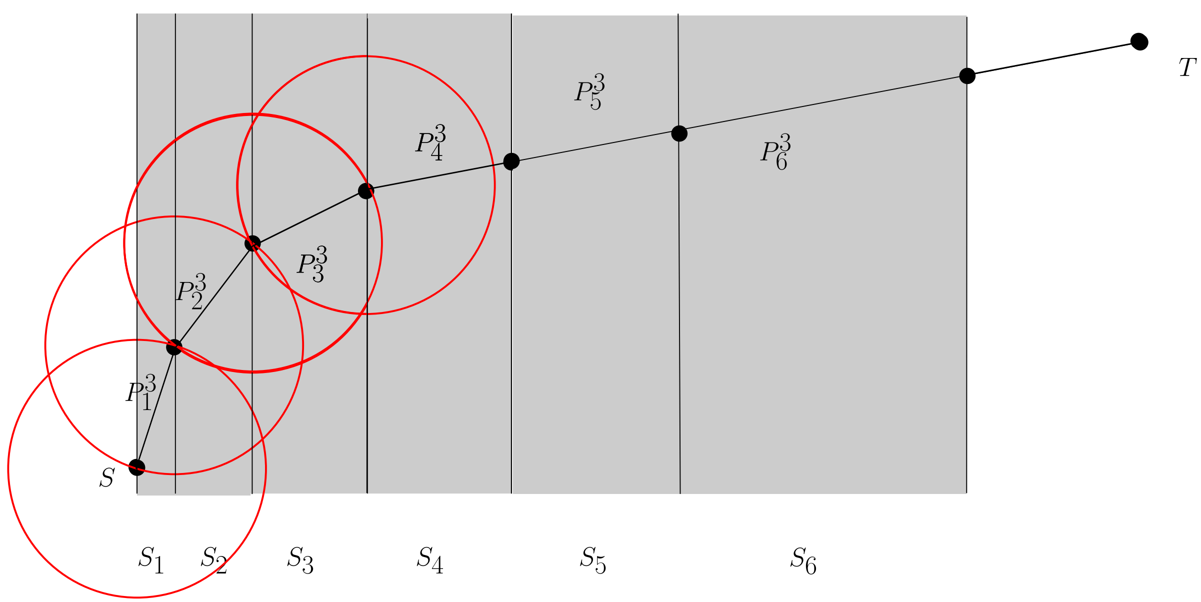

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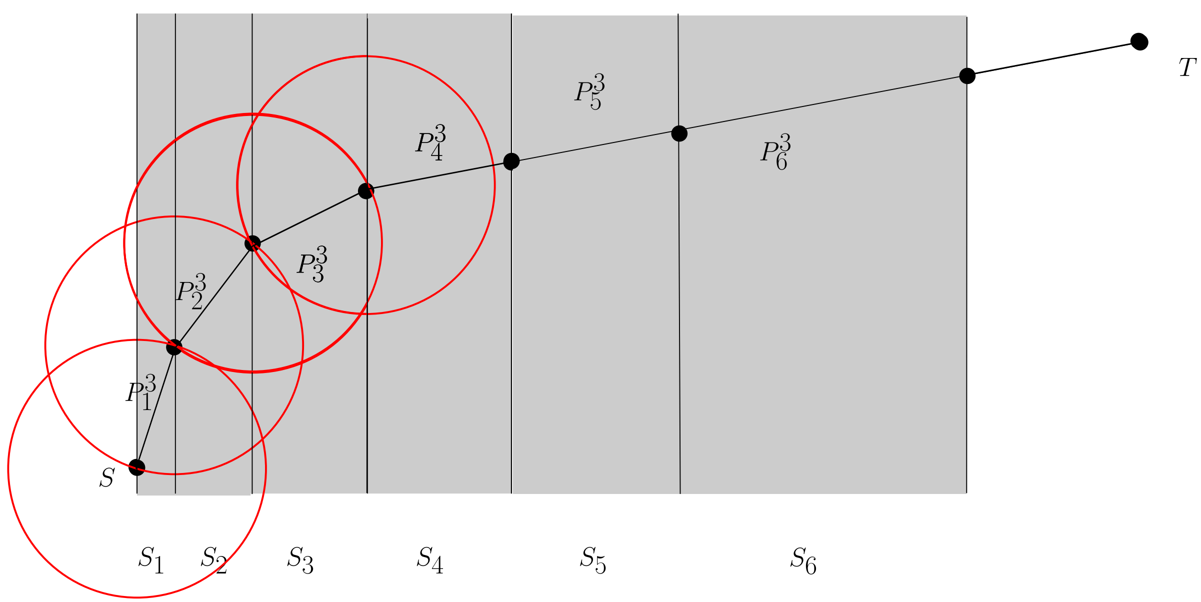

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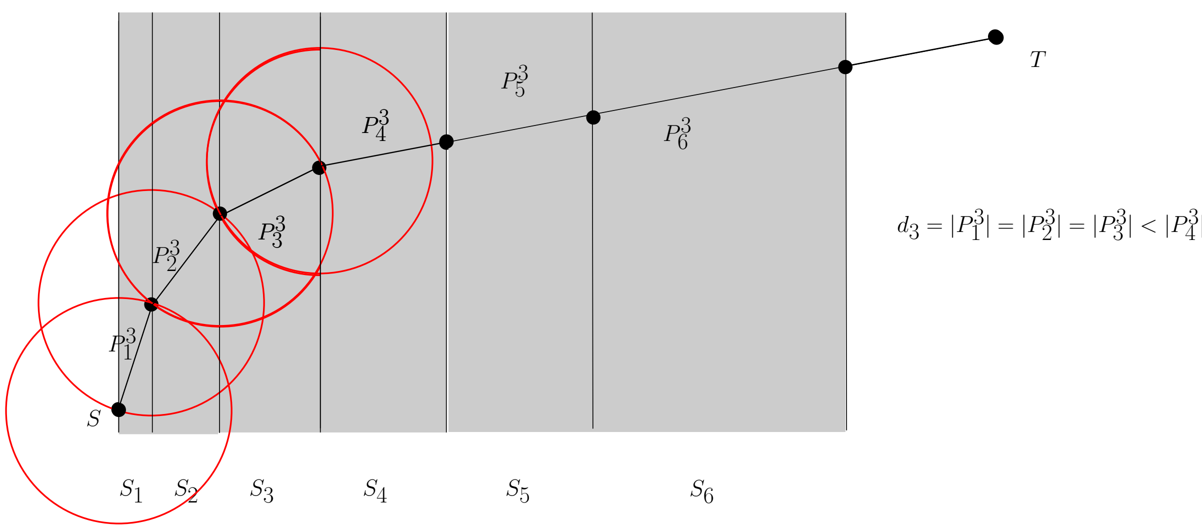
Efficient algorithm



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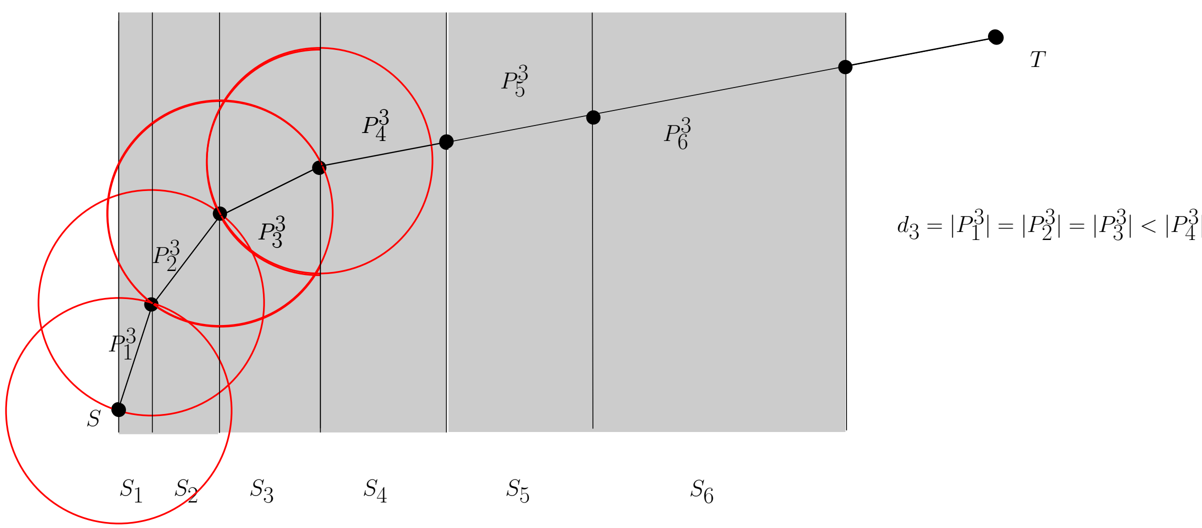

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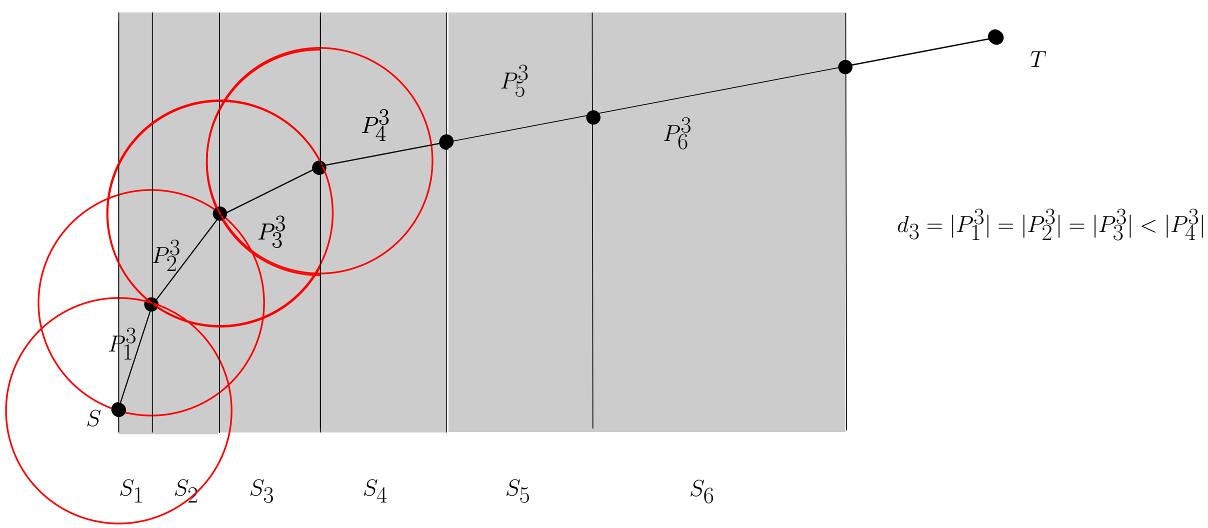

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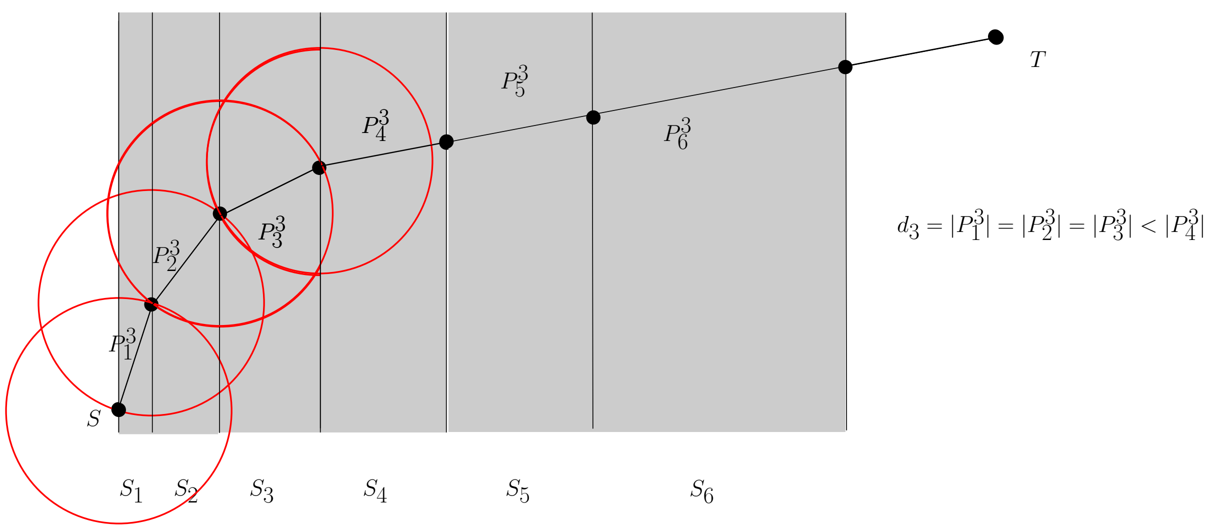

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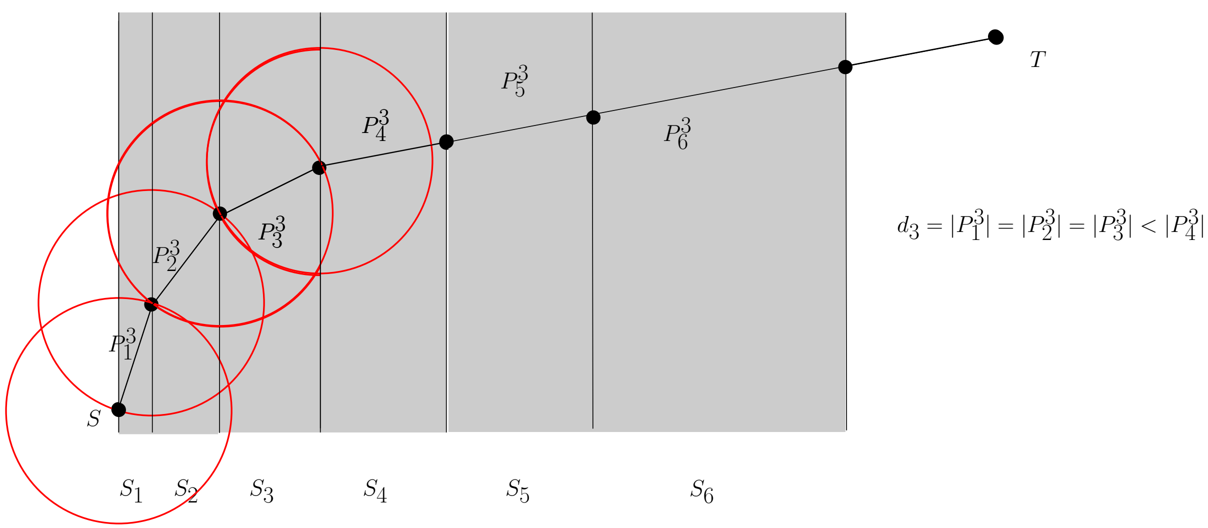

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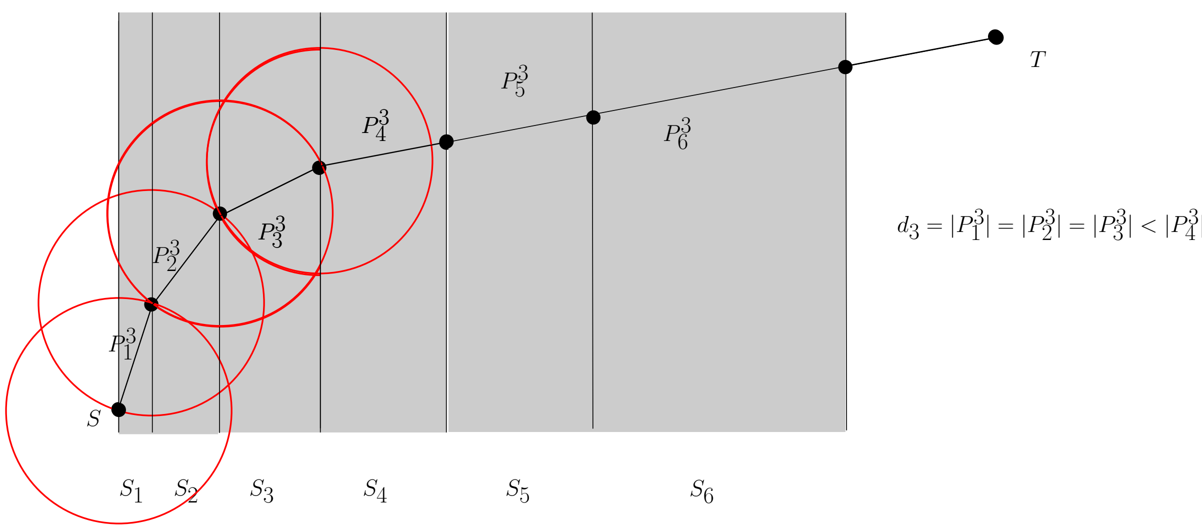

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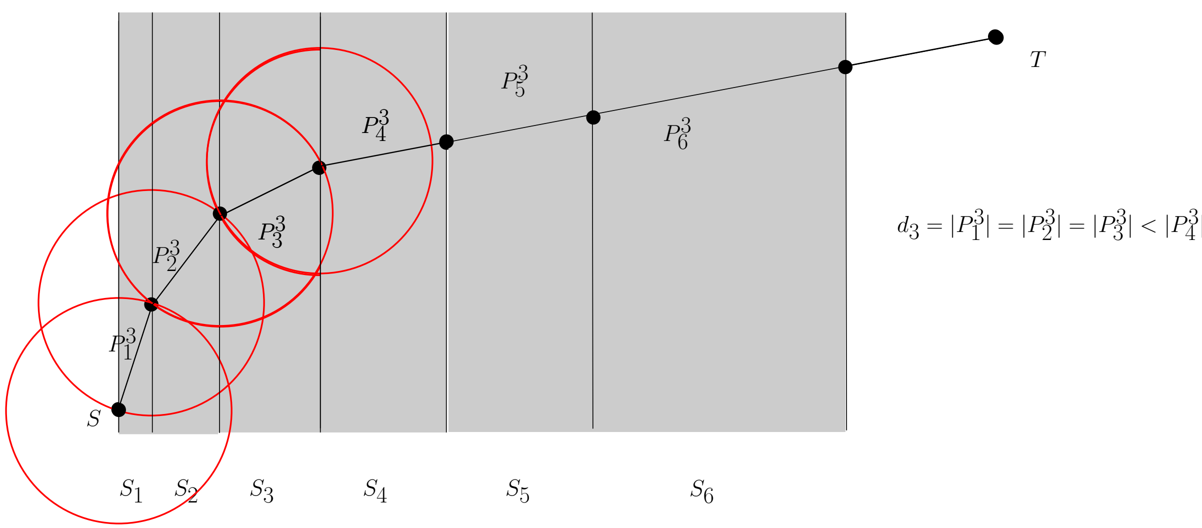
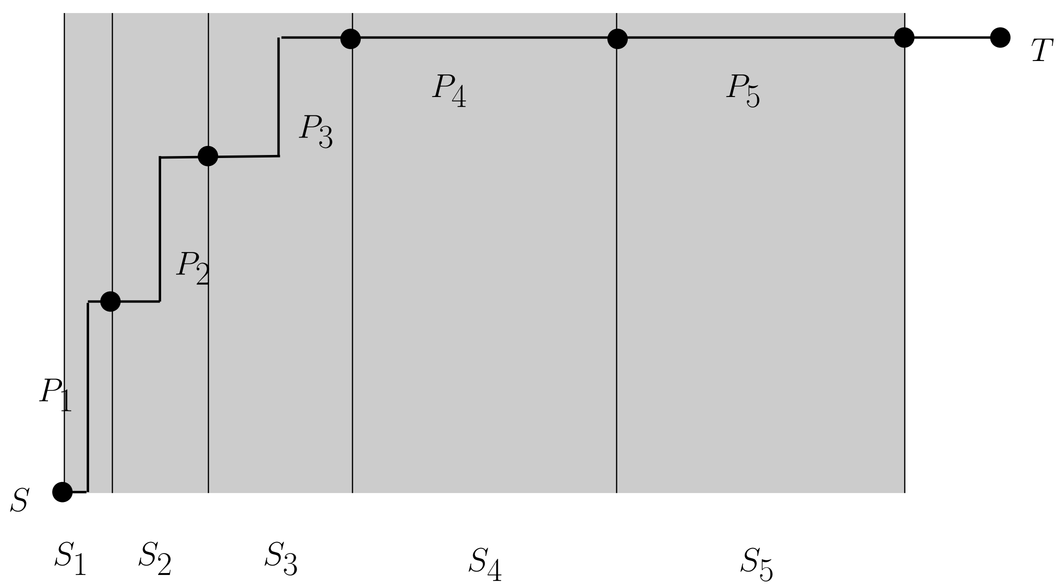


L_1: Incrementally



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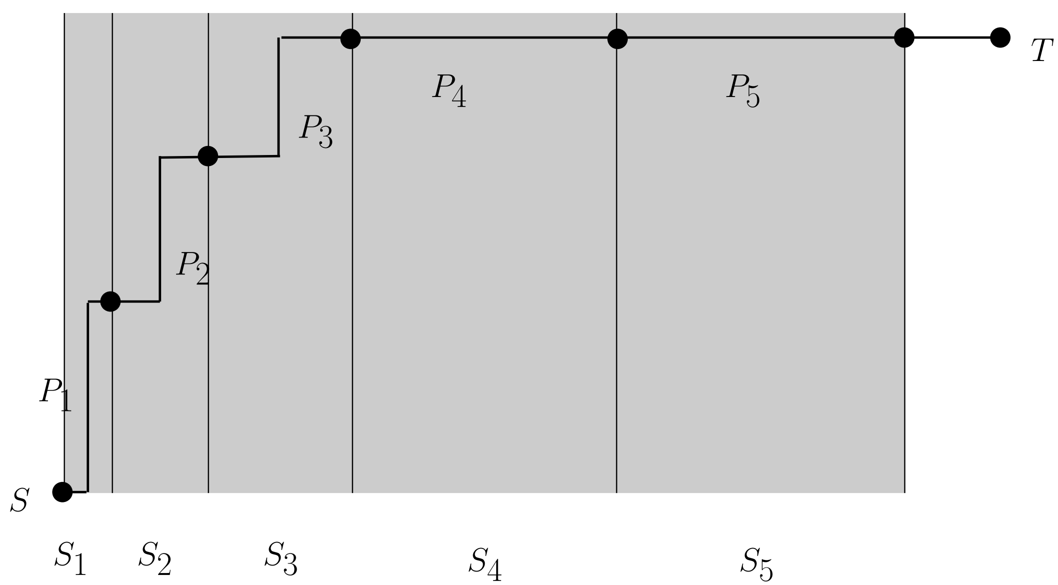

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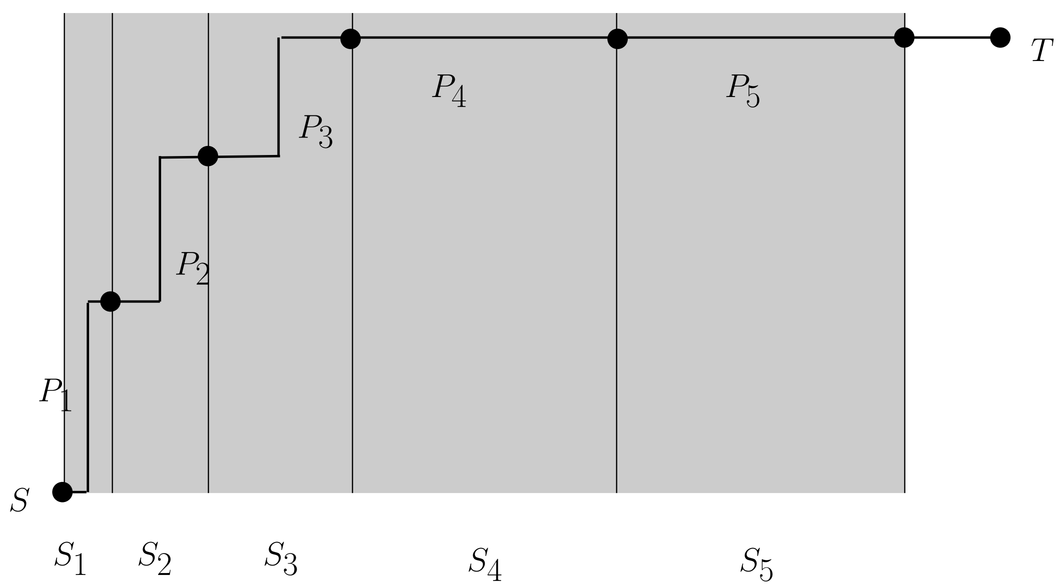

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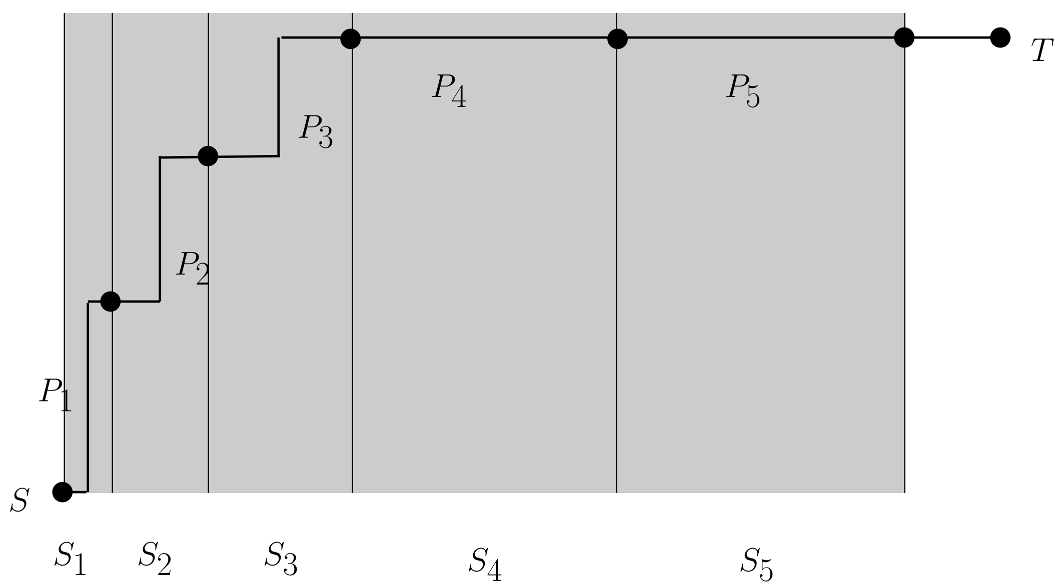

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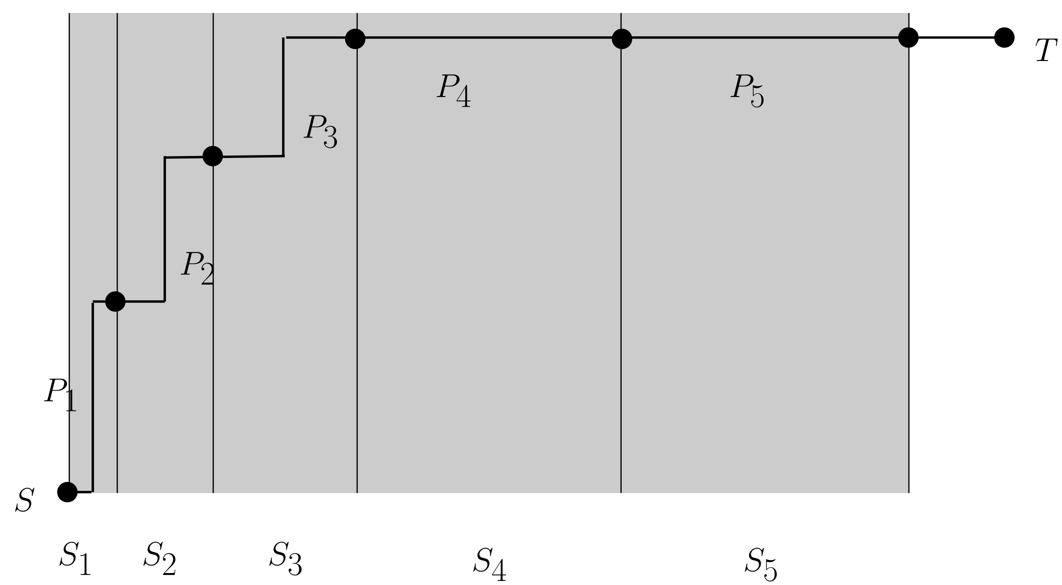

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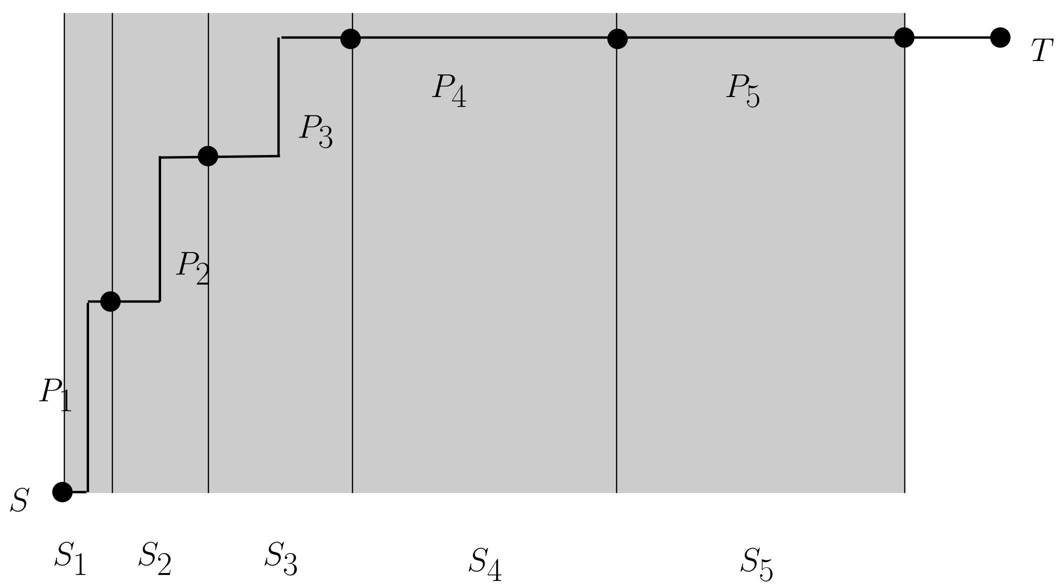

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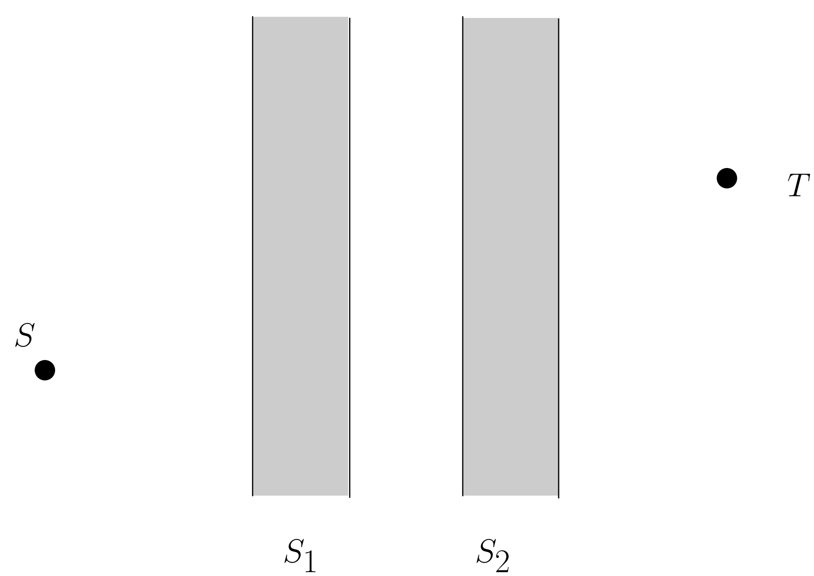

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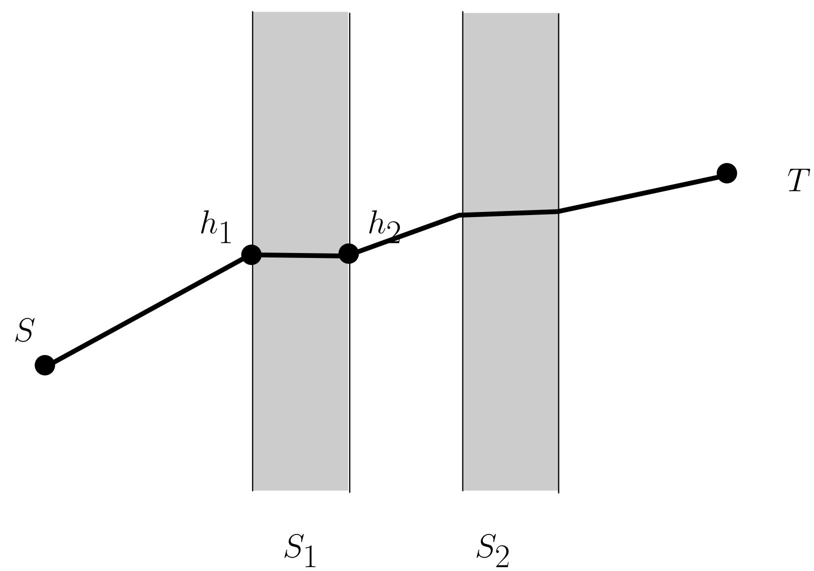

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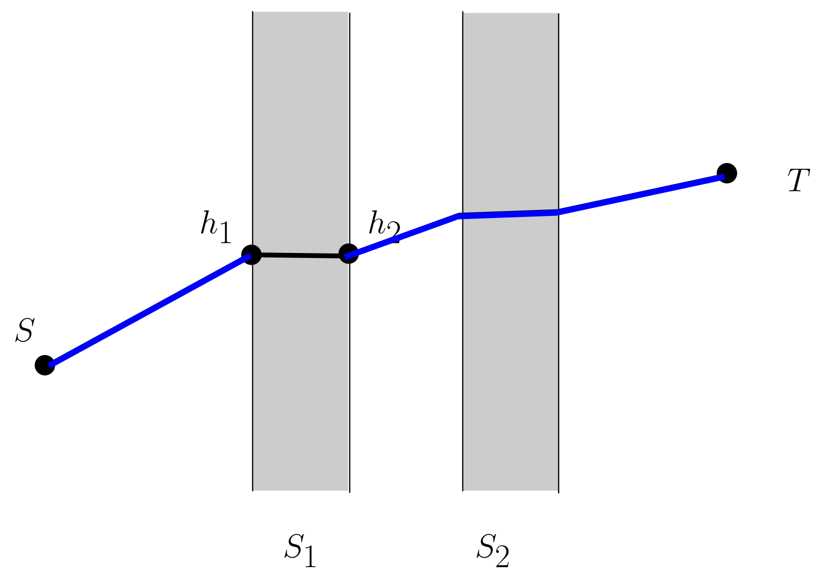

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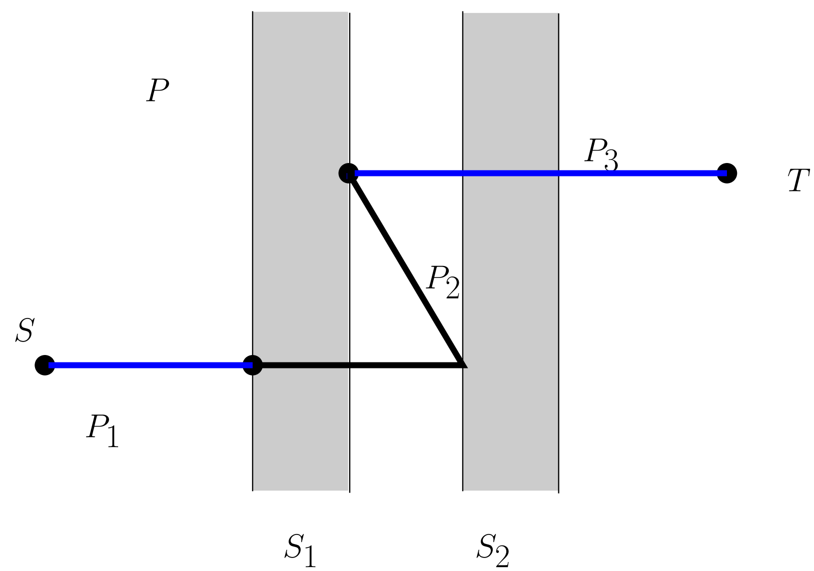

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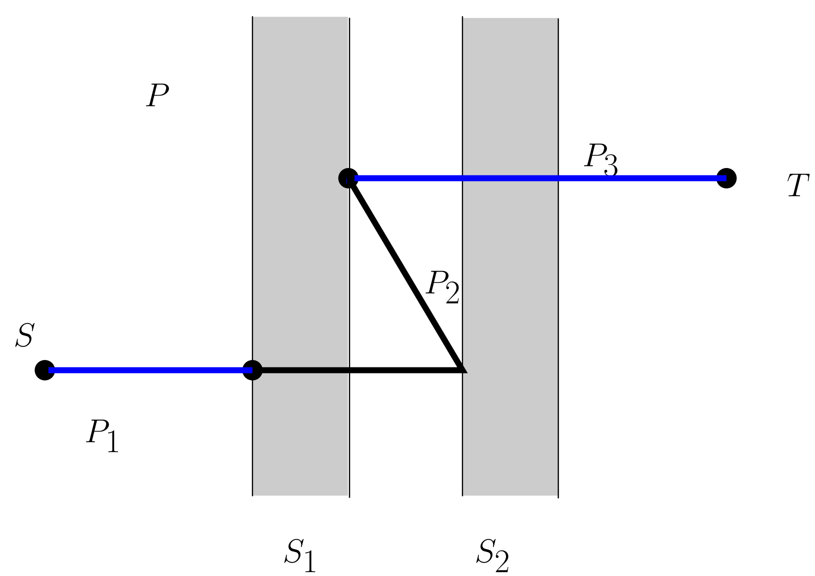
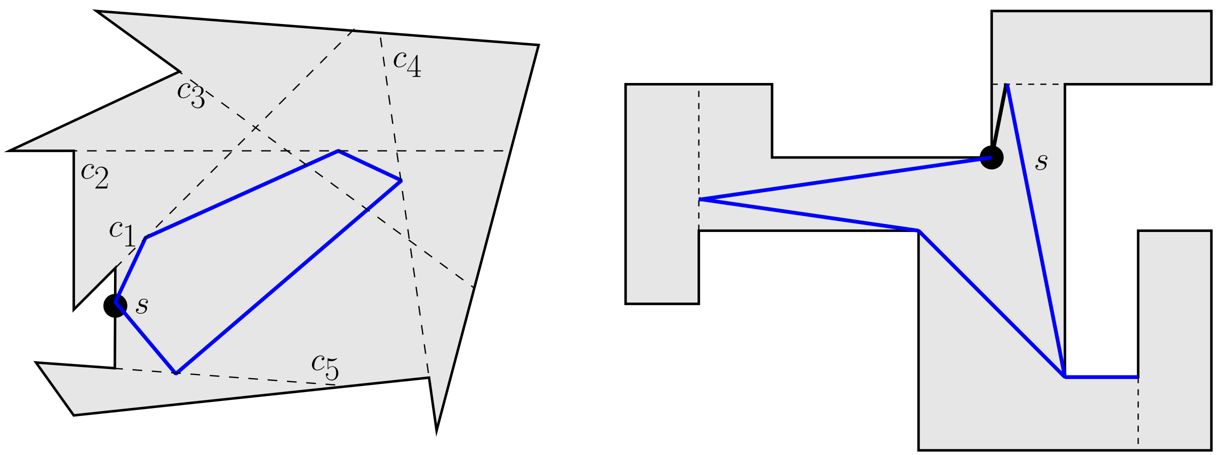


SWR=SIR conjecture



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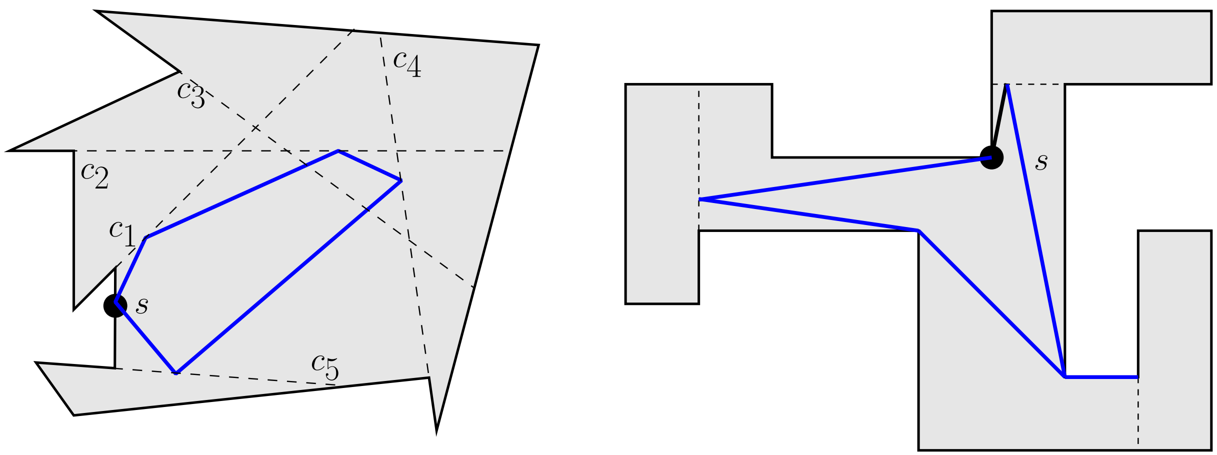

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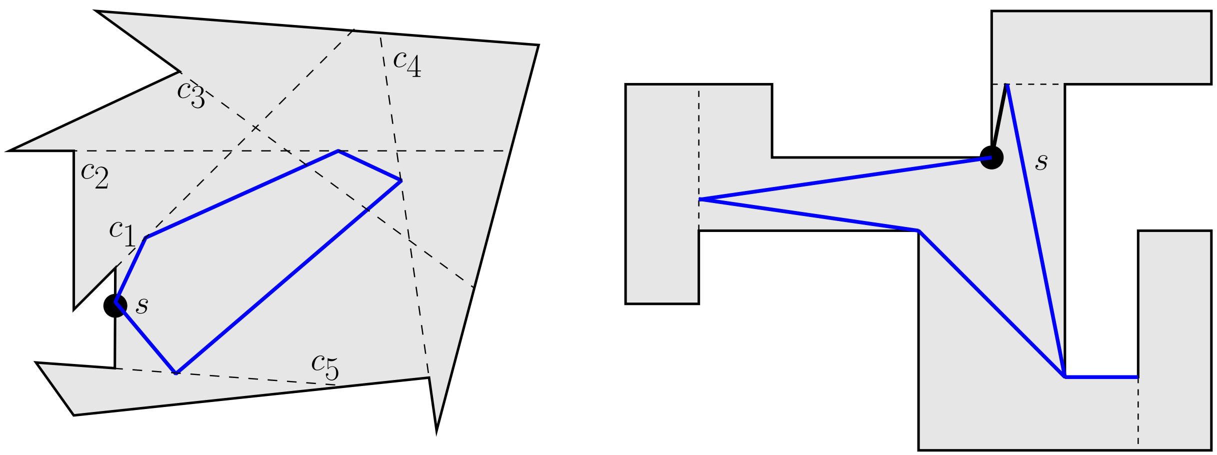

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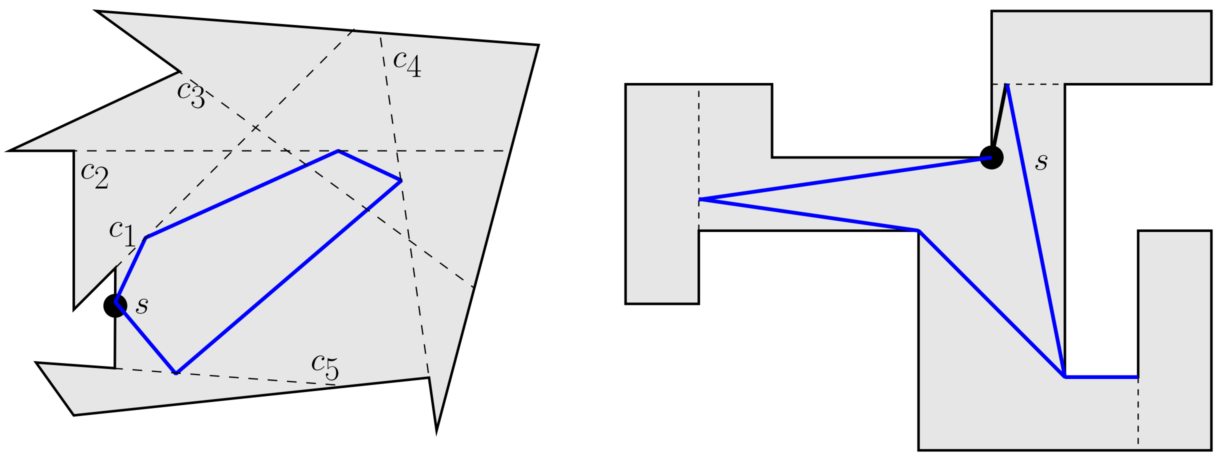

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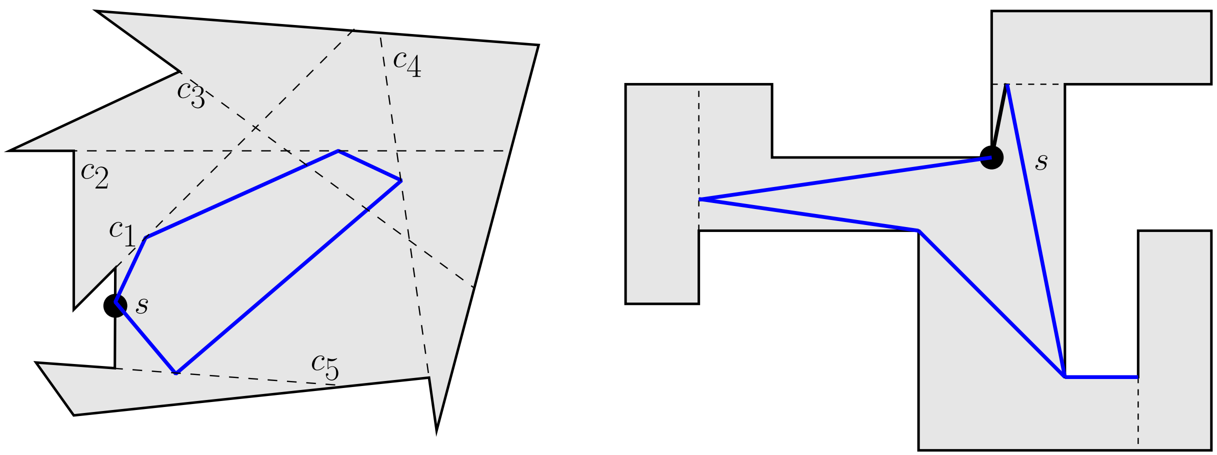

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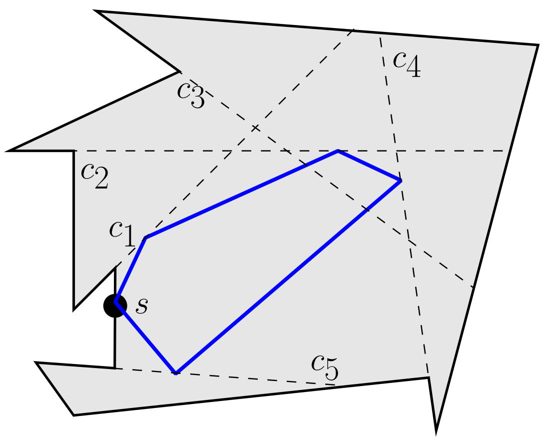

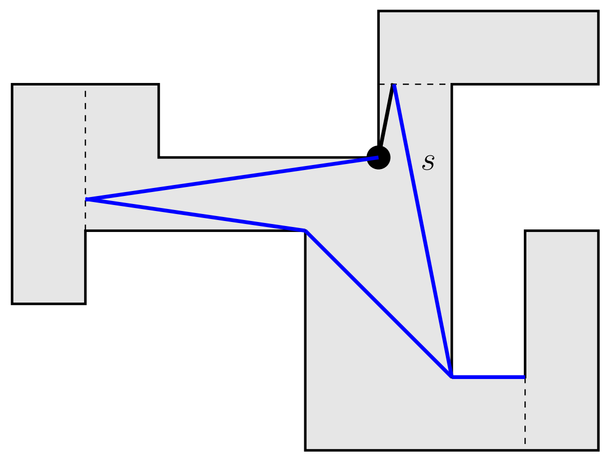

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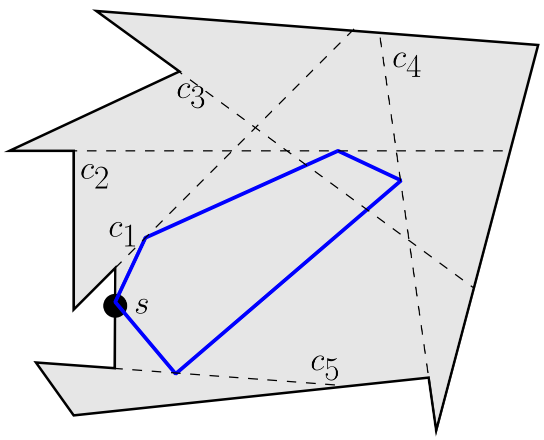

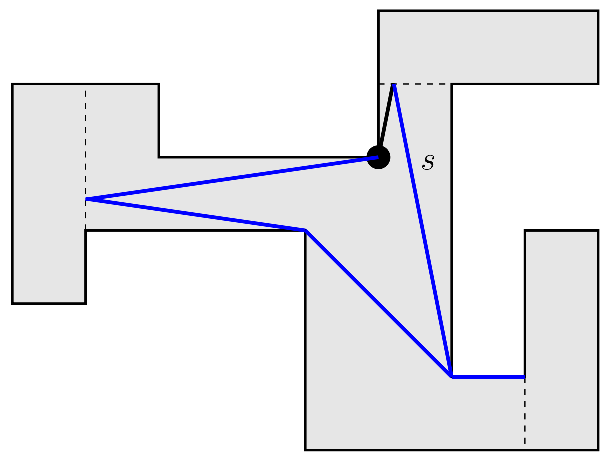

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