Inspecting a set of strips optimally

Tom Kamphans and Elmar Langetepe
University of Braunschweig and University of Bonn

Model: General inspection paths

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- Polygons in the plane,

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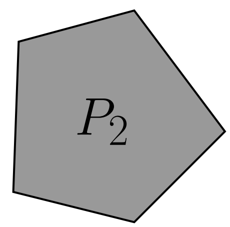

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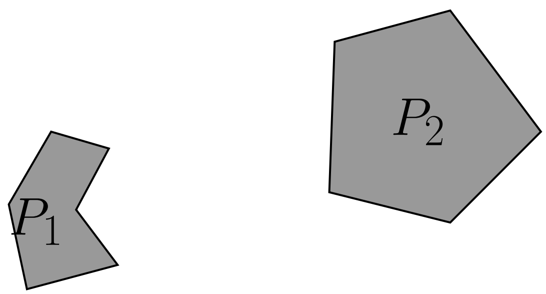

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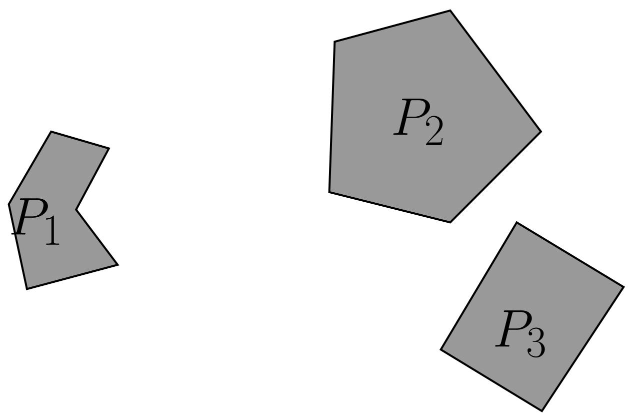

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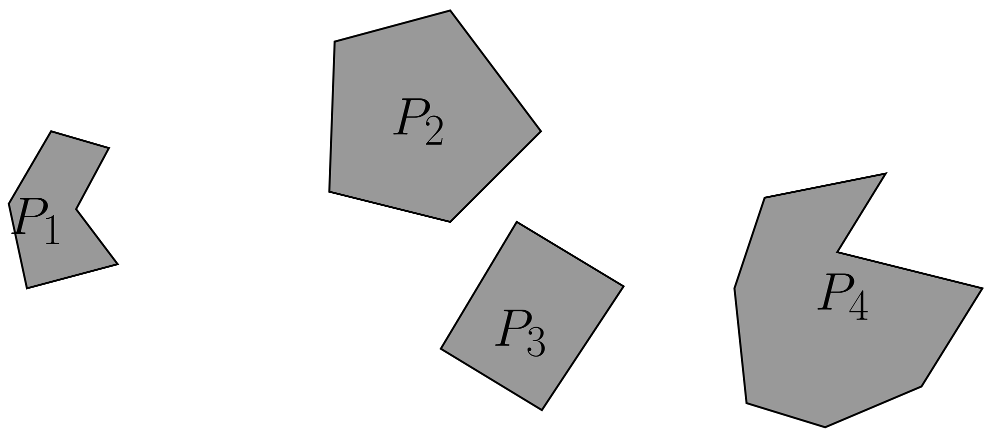

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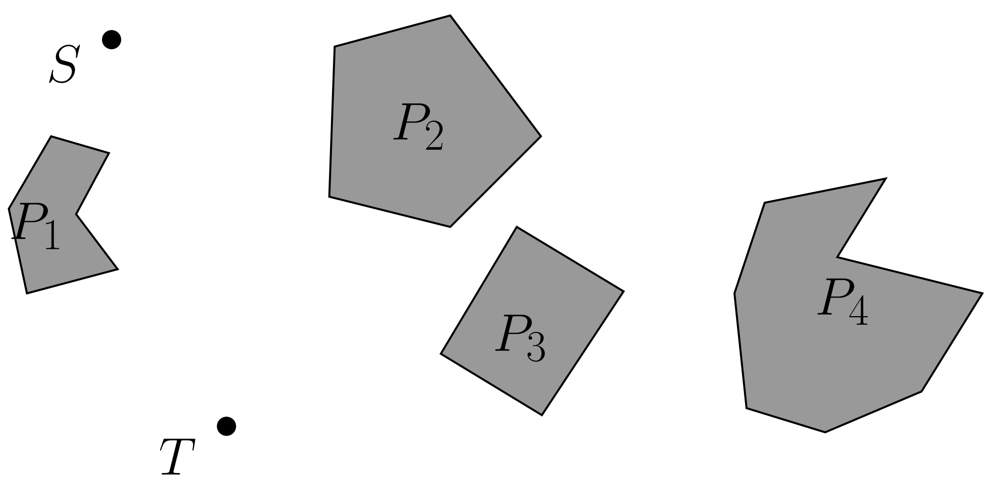

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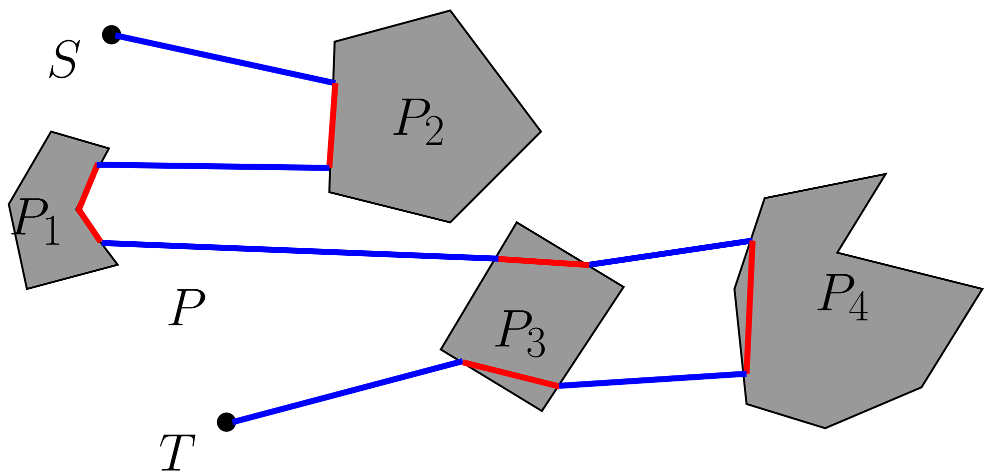

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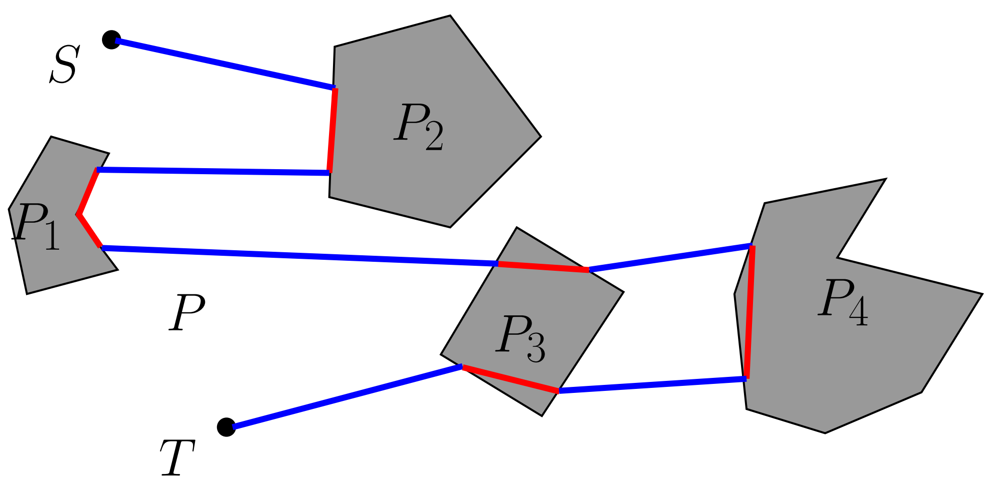

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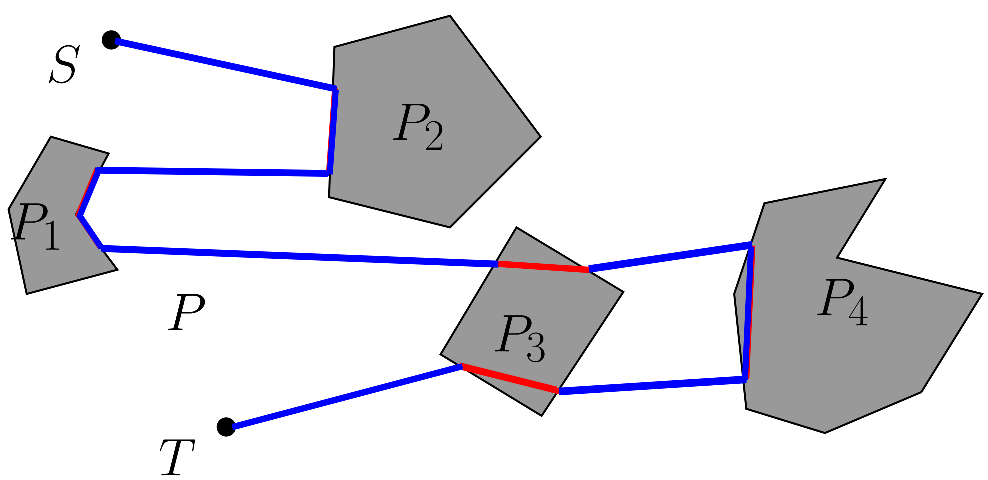

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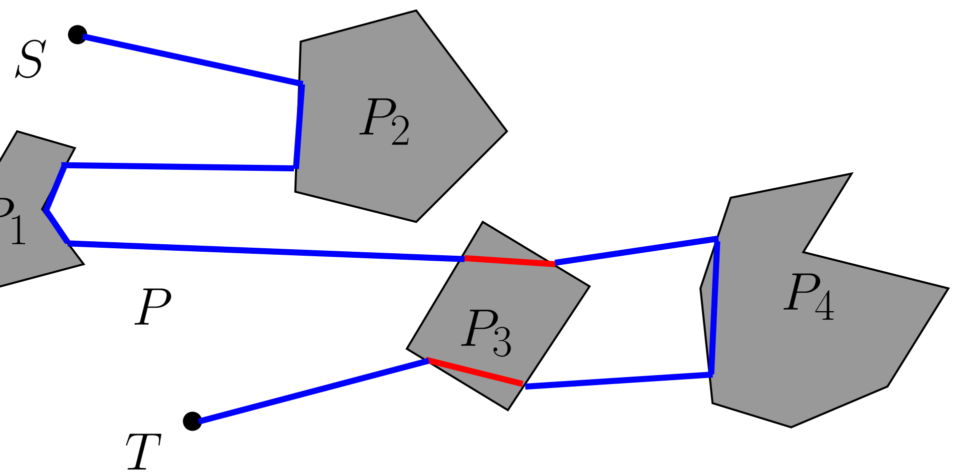

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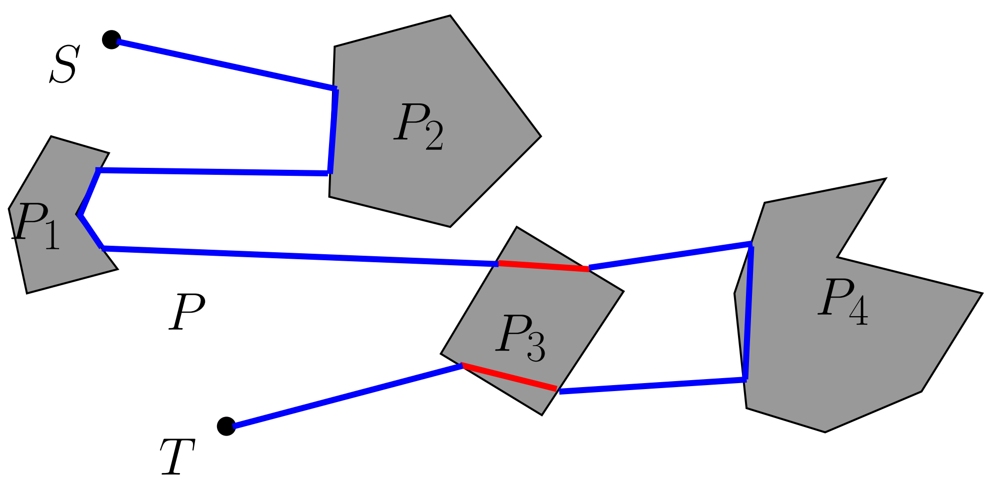

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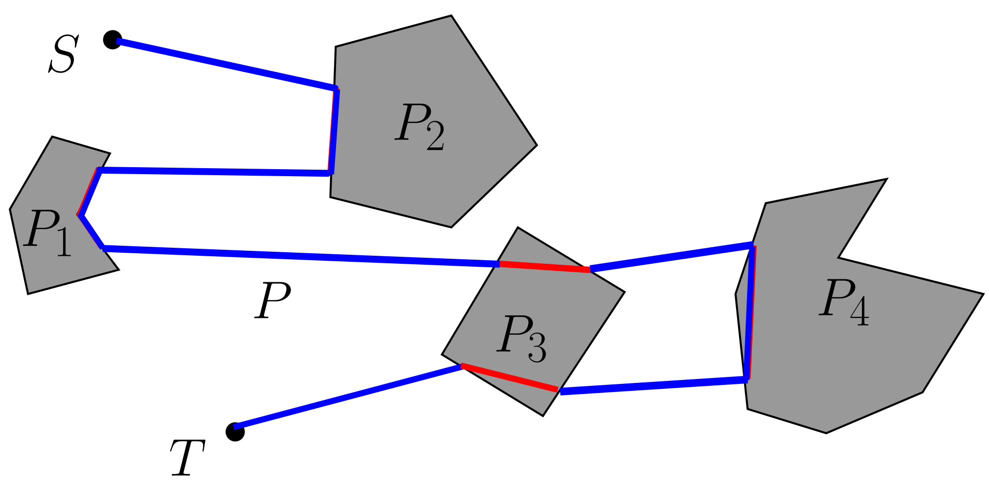

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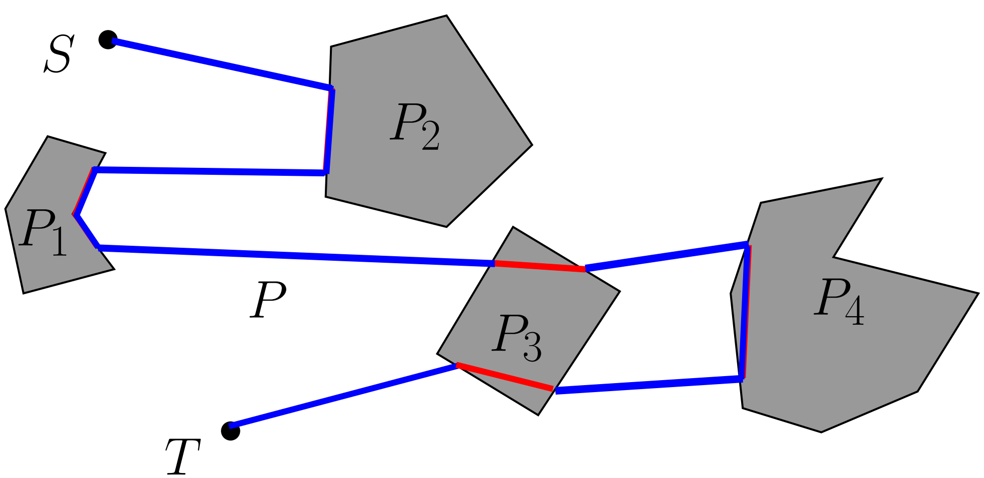

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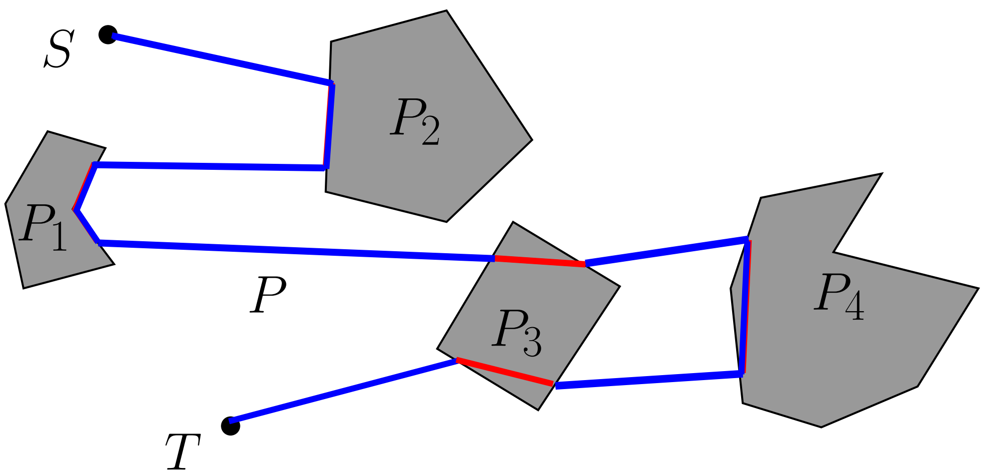

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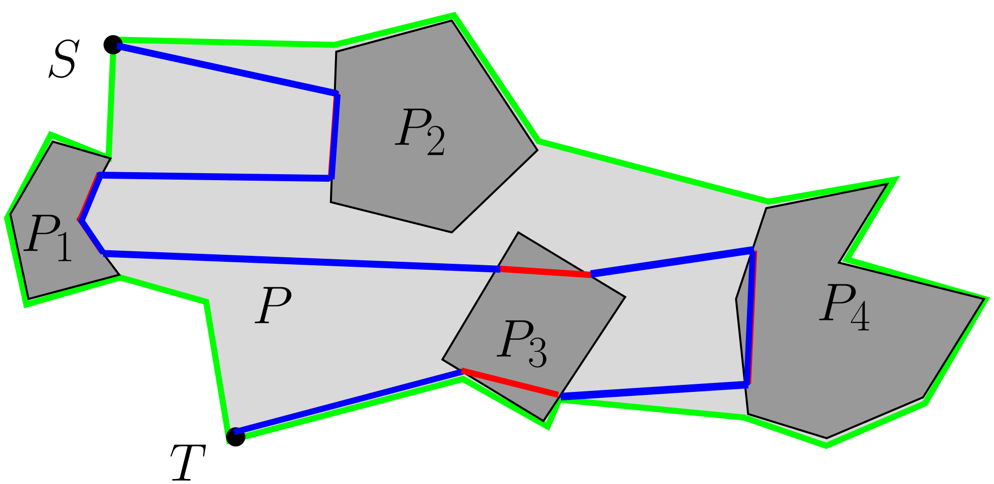

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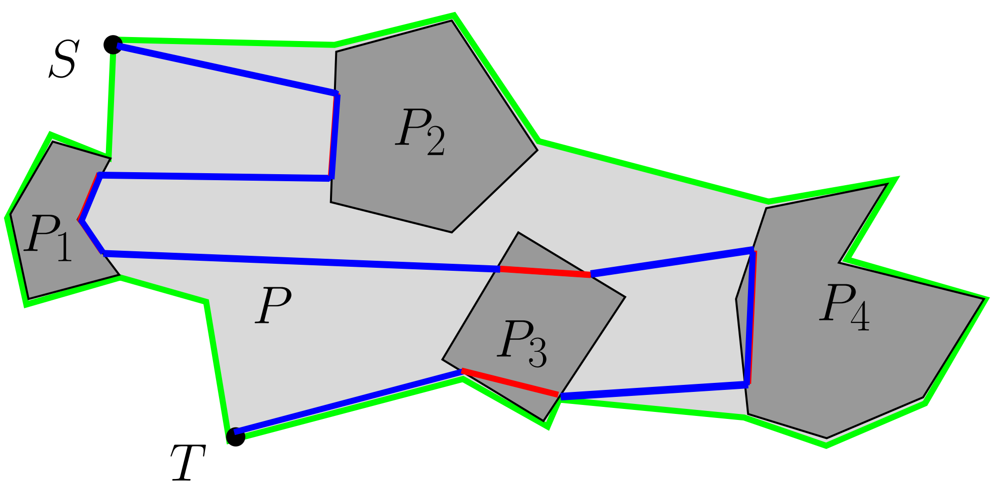
- By construction: Polygons on the boundary of one outer polygon?
- Order along boundary? Multiple visits?



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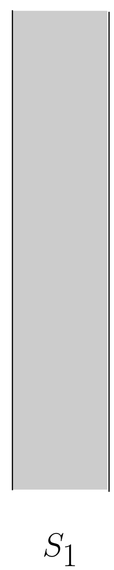
- Strips $S_1, S_2, \ldots, S_n$ of different widths $w_1, \ldots, w_n$



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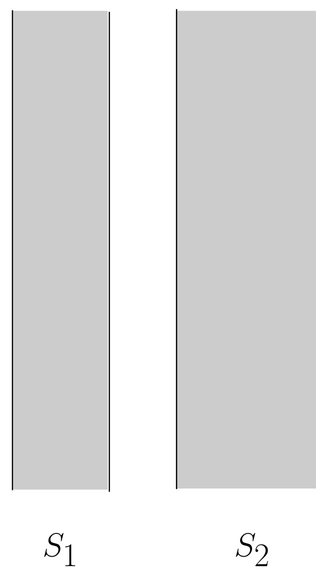

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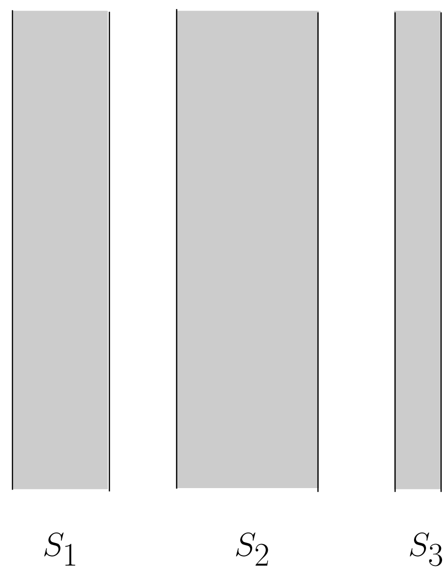

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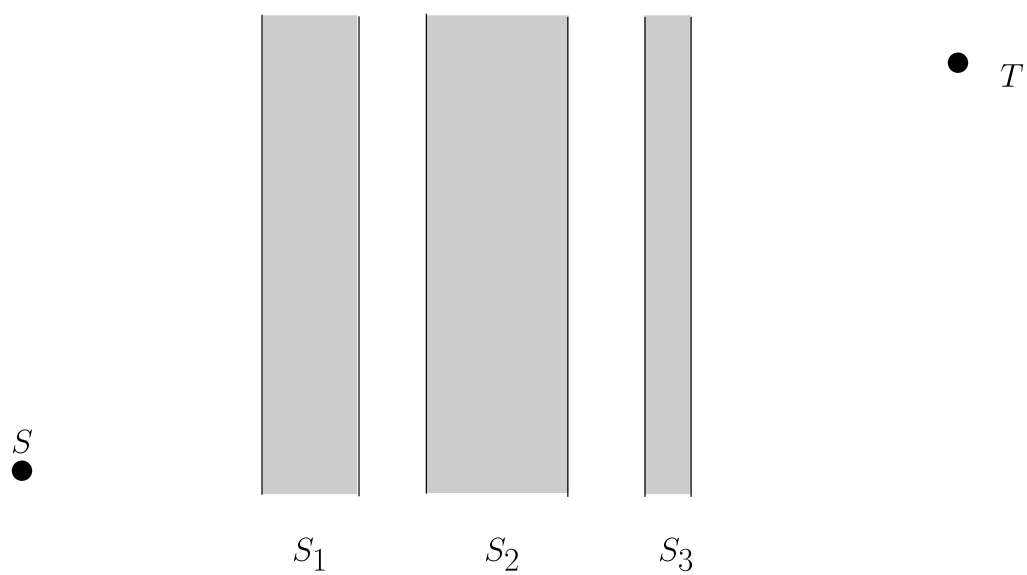

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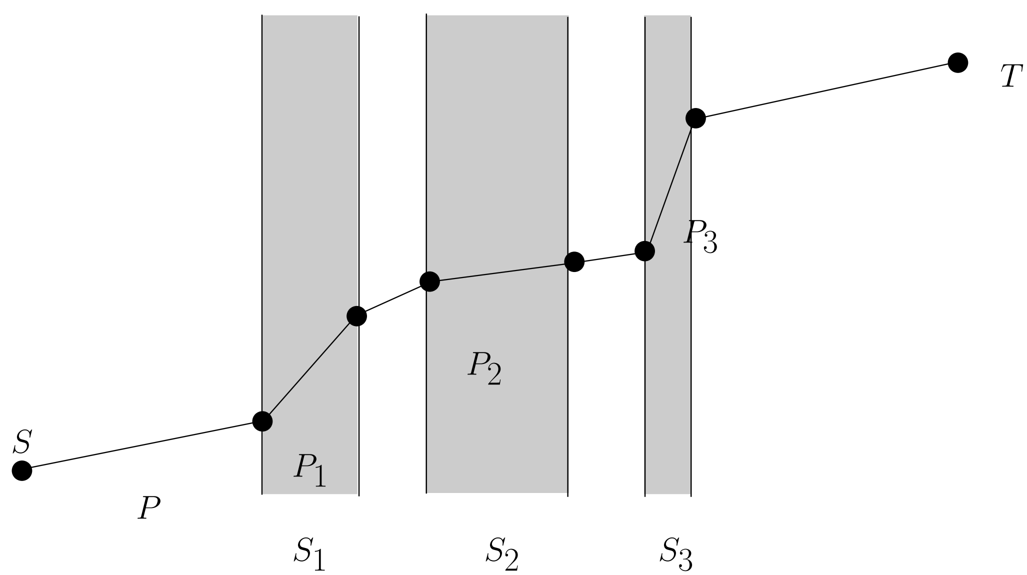

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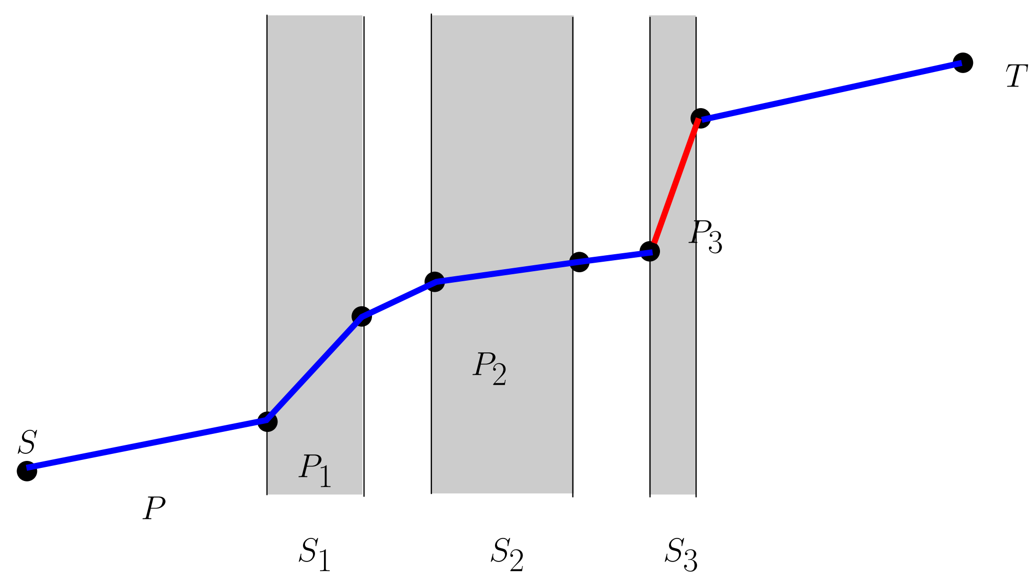

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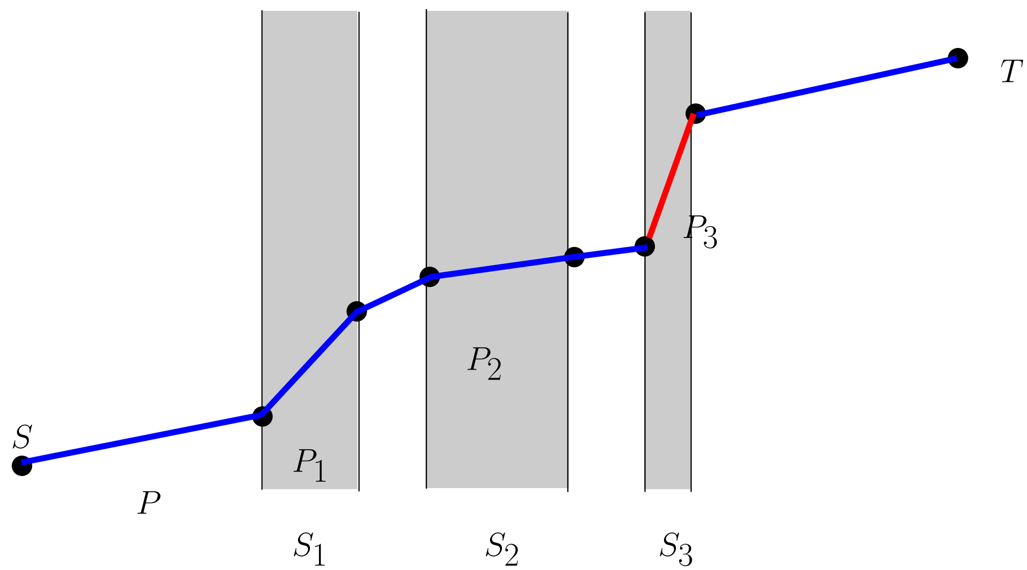

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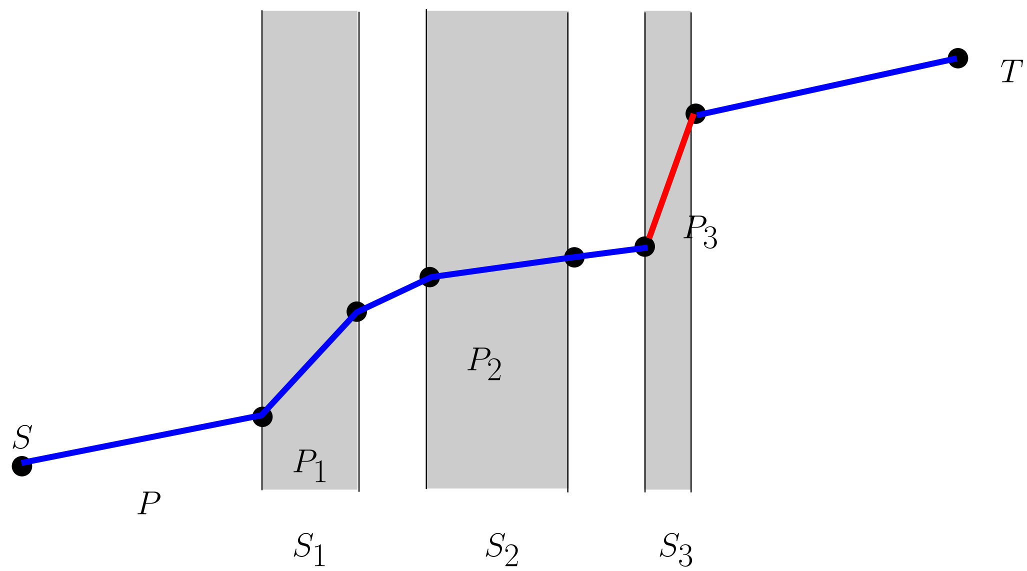

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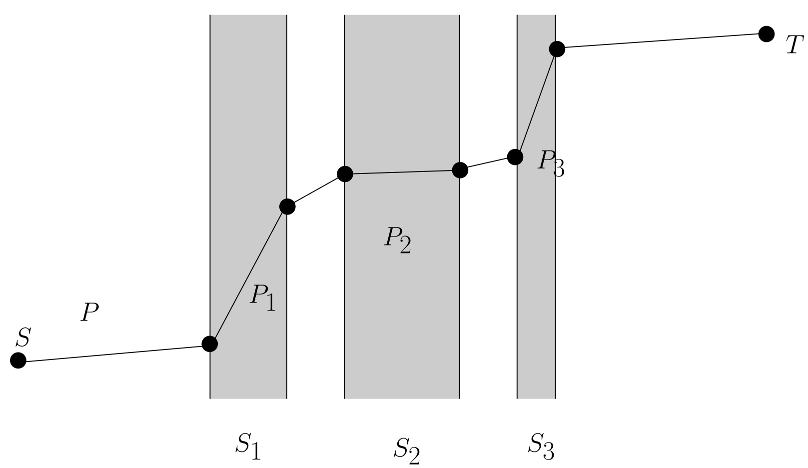
- Structural properties, reordering: Strips by increasing widths



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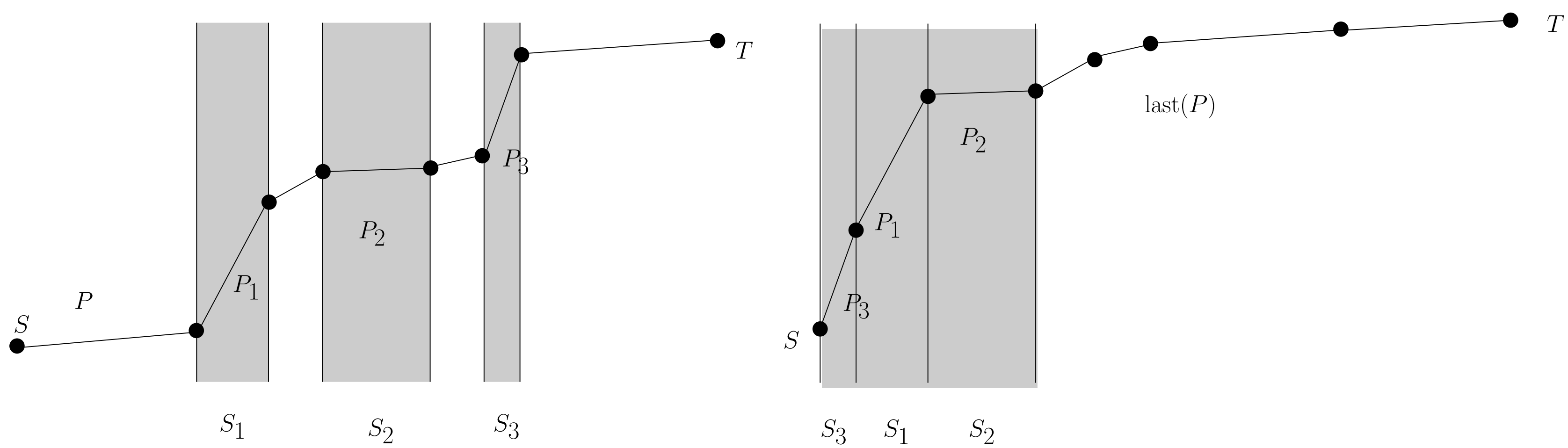

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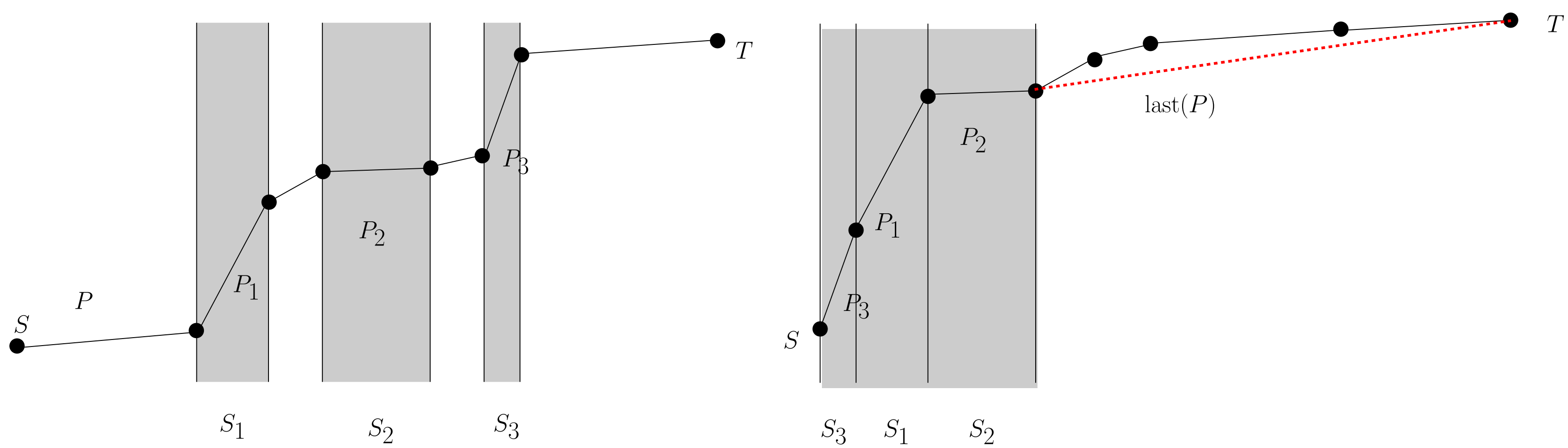

Inspection of a set of strips

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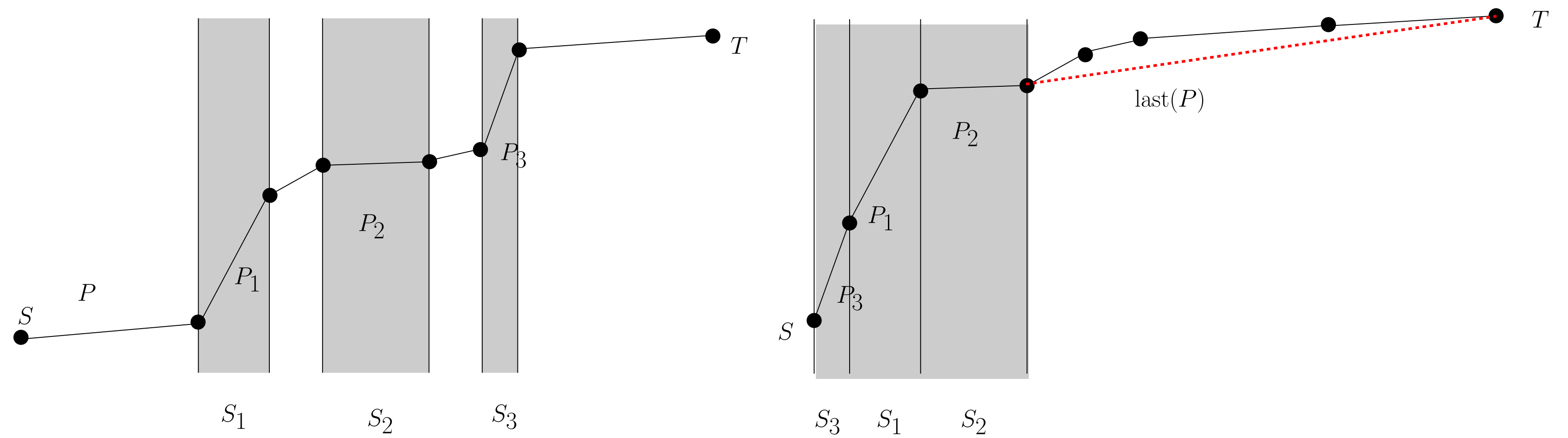
- Lemma: Kinks at the boundaries, the same slope between two successive strips



Inspection of a set of strips

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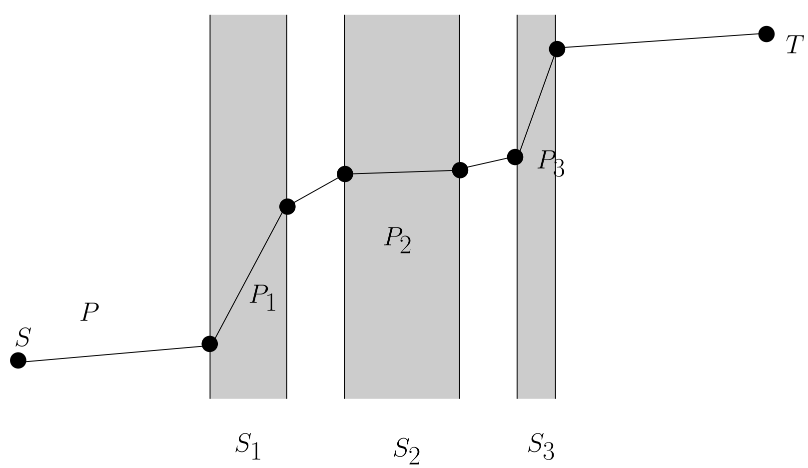
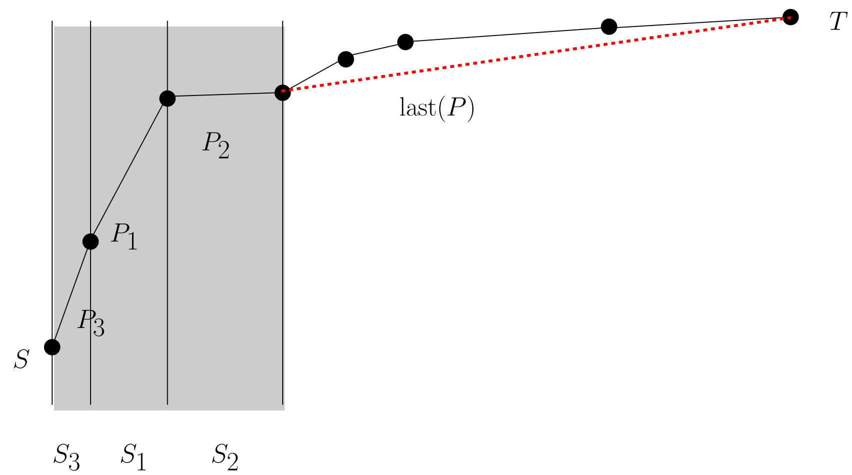
- Lemma: Kinks at the boundaries, the same slope between two successive strips
- In the following: n strips sorted by widths $w_i \leq w_{i+1}$ for $i = 1, \ldots, n-1$





Structural properties: Relevant strips

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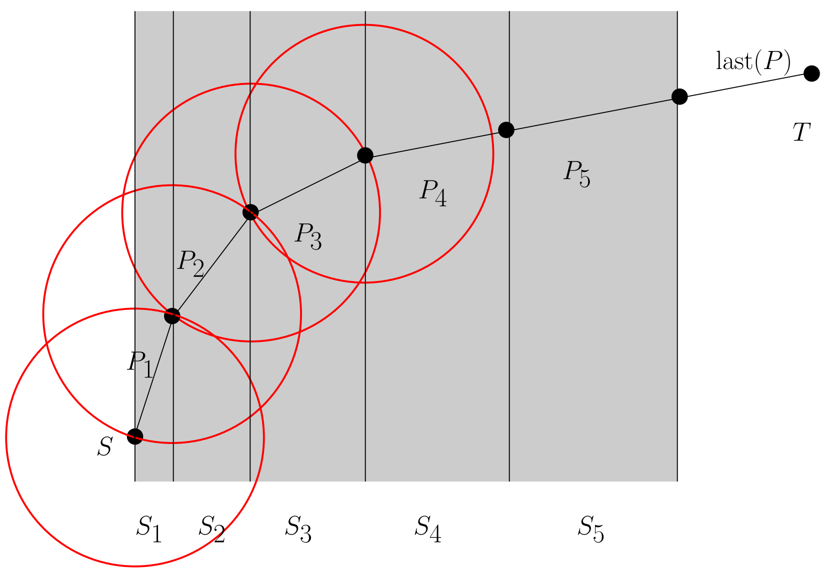
- Lemma: For optimal solution $\exists K \leq n$: $|P_i| = d$ for $i = 1, \ldots K$, $|P_i| > d$ for $i = K+1, \ldots n$, konvex wrt. ST



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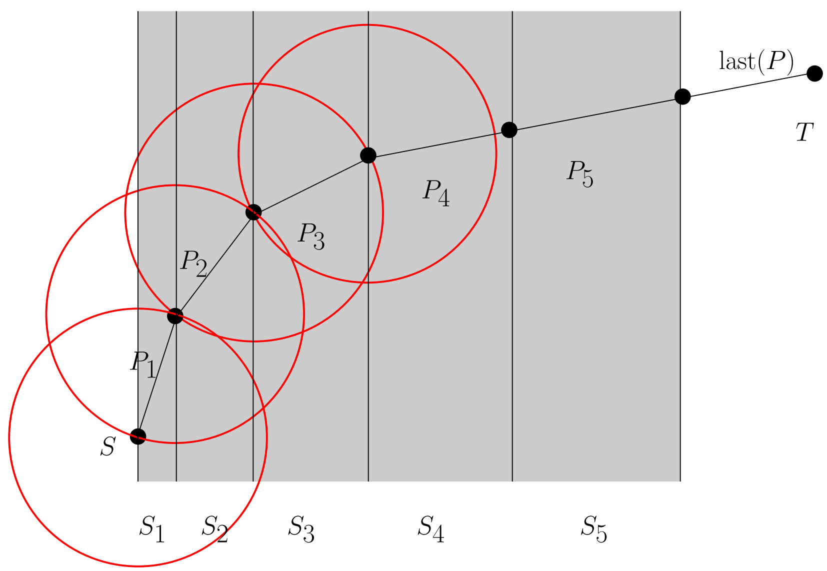

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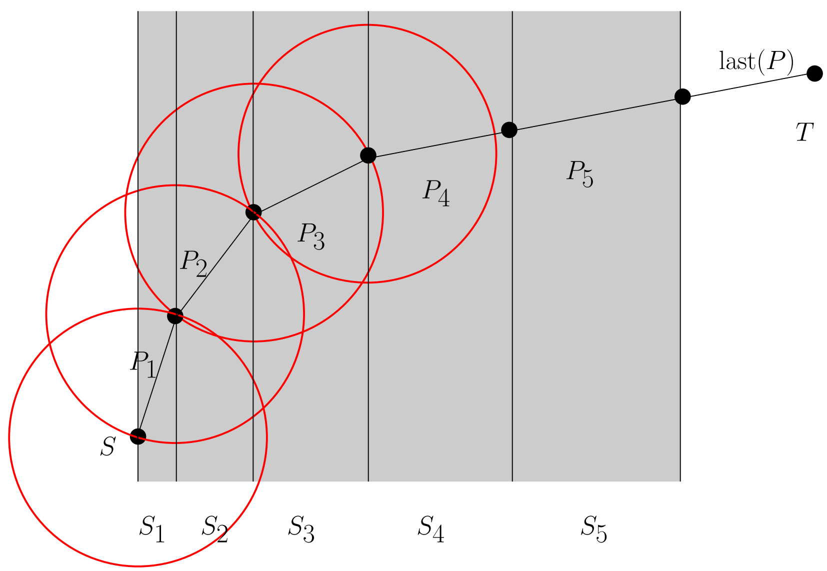

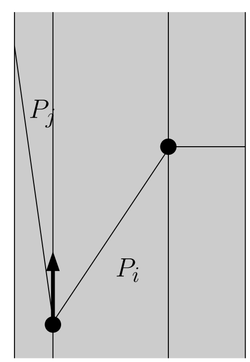

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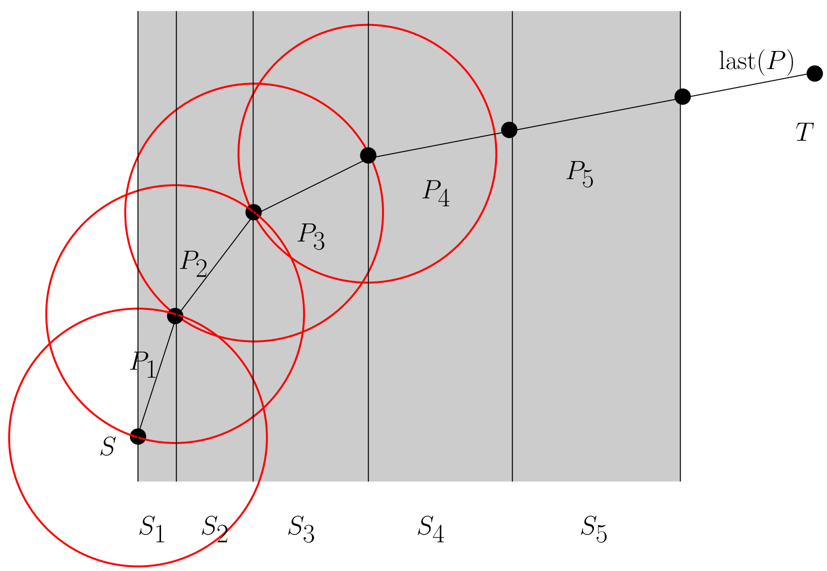
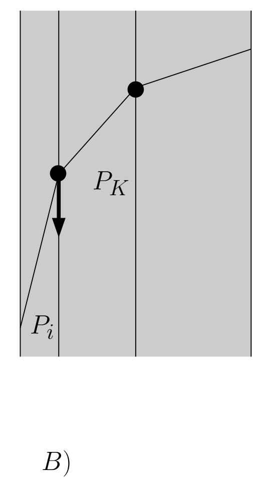
- B) Ind. K s.th. P_K opt.: $|P_i| > |P_K|$, $w_i \leq w_K$ is impossible





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- A) Optimal path is monotonically increasing
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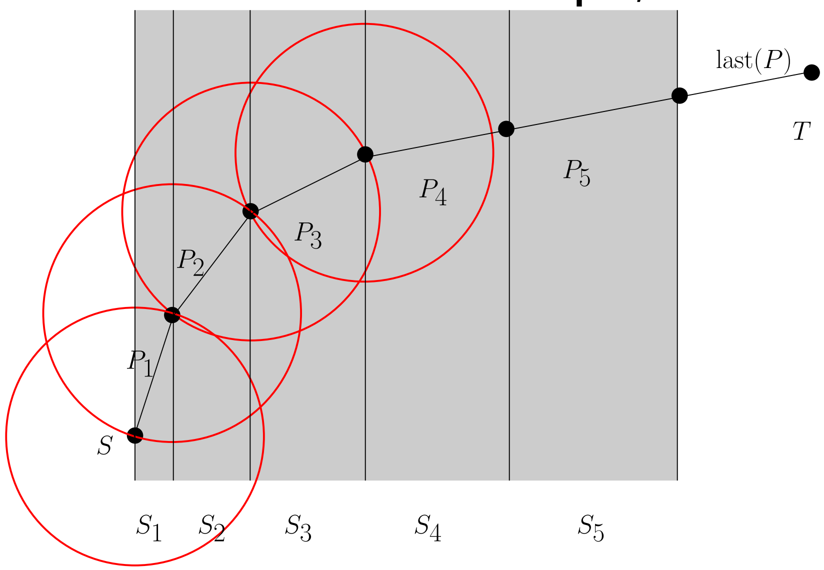
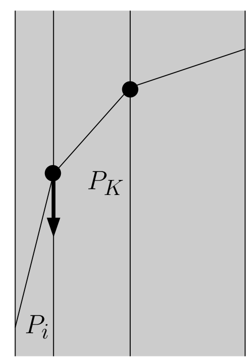
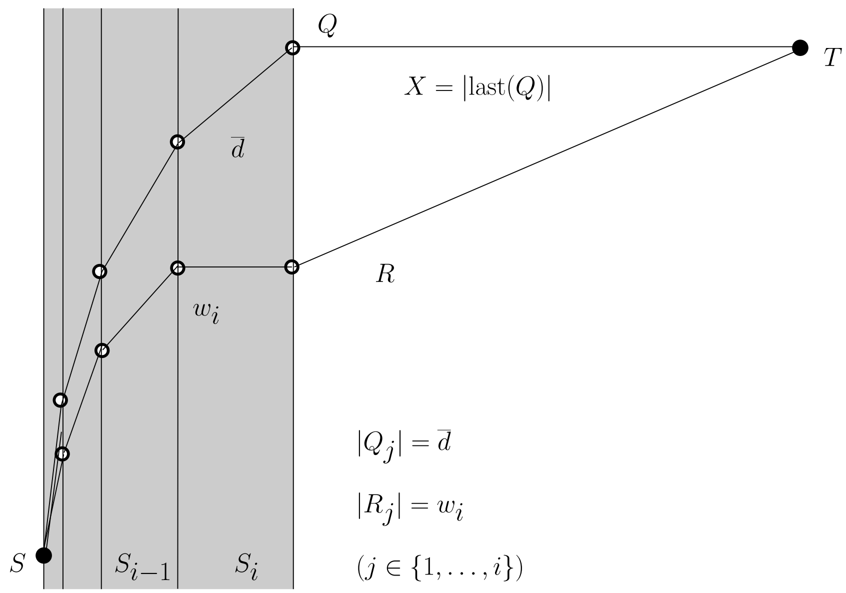
- Lemma: Succ. add strips, index K does never decrease





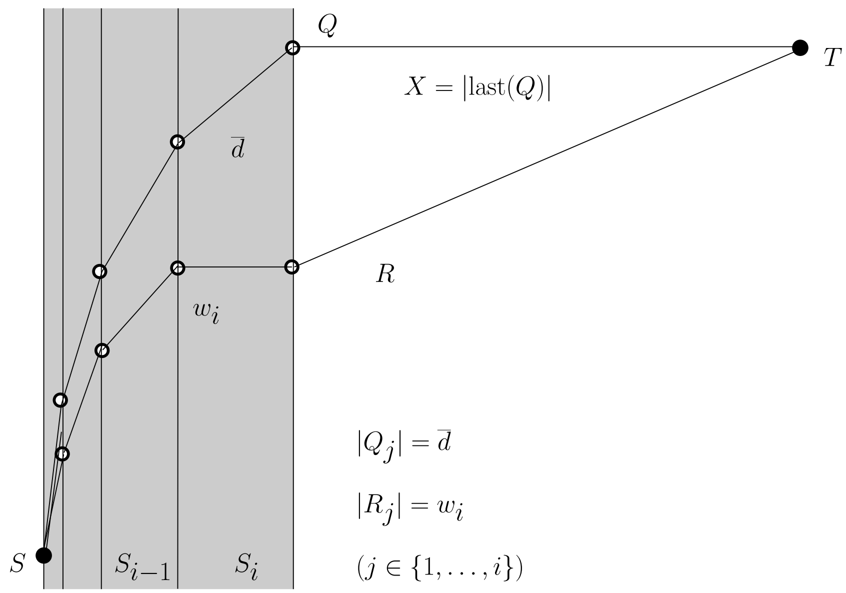
B)

Optimizing for given index $K = i$



Optimizing for given index $K = i$

- Find best solution for $K = i$: $|P_j| = d$ for $j = 1, \ldots i$



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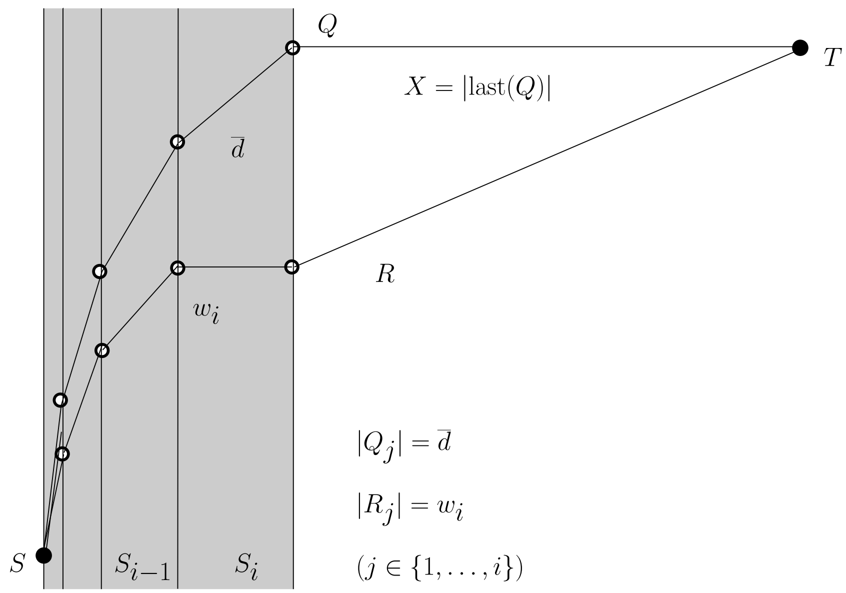
- Start with $d = w_i$, minimize function $f_i(d)$



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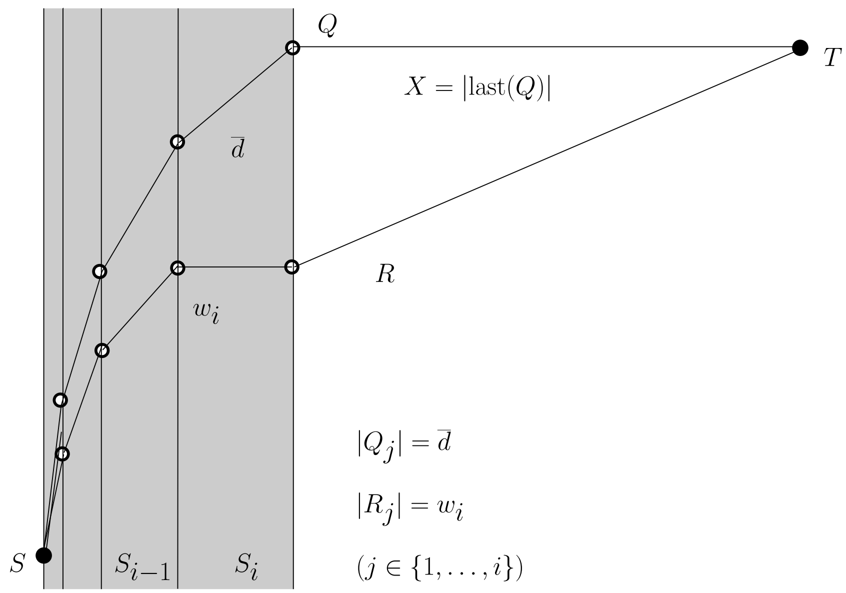
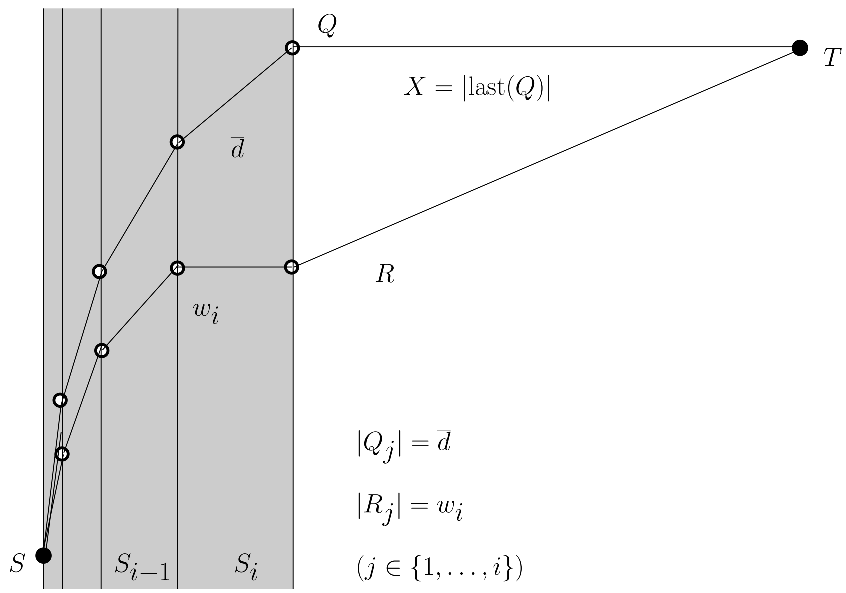
- $\min_d f_i(d) := d \times (i-1) + \sqrt{X^2 + \left(t_y - \sum_{j=1}^{i} \sqrt{d^2 - w_j^2}\right)^2}$



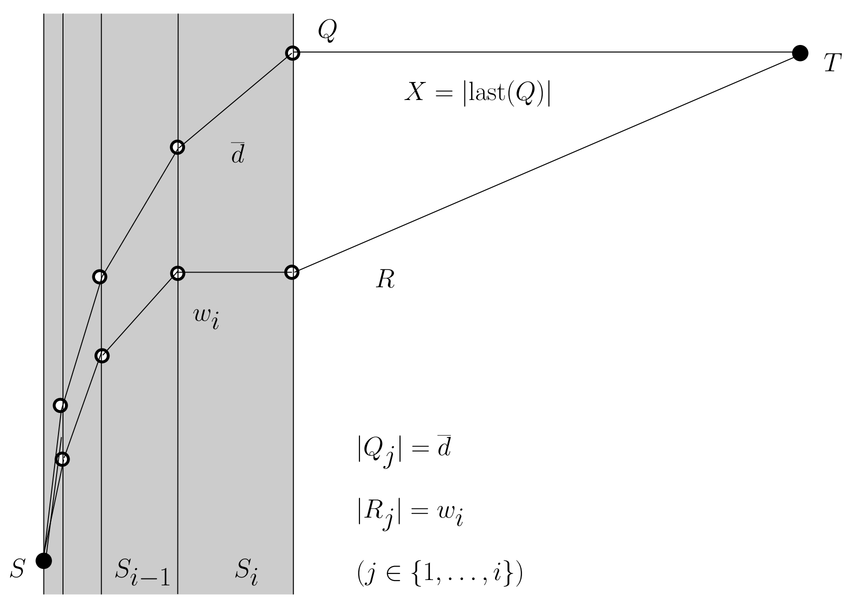
Optimal d_i with $|P_j| = d_i$ for $j = 1, \ldots i$



Optimal d_i with $|P_j| = d_i$ for $j = 1, \ldots i$

- Solve:

$$f_i'(d) = i - 1 - \frac{\left(t_y - \sum_{j=1}^{i} \sqrt{d^2 - w_j^2}\right)}{\sqrt{X^2 + \left(t_y - \sum_{j=1}^{i} \sqrt{d^2 - w_j^2}\right)^2}} \cdot \left(\sum_{j=1}^{i} \frac{d}{\sqrt{d^2 - w_j^2}}\right)$$



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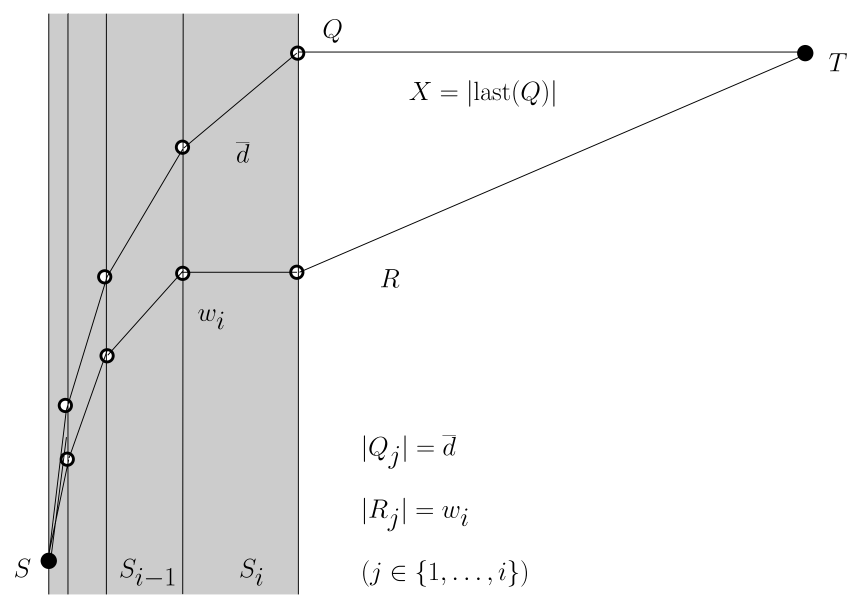
- Analysis: $f_i'(d)$ is monotone, one minimum at d_i for every i



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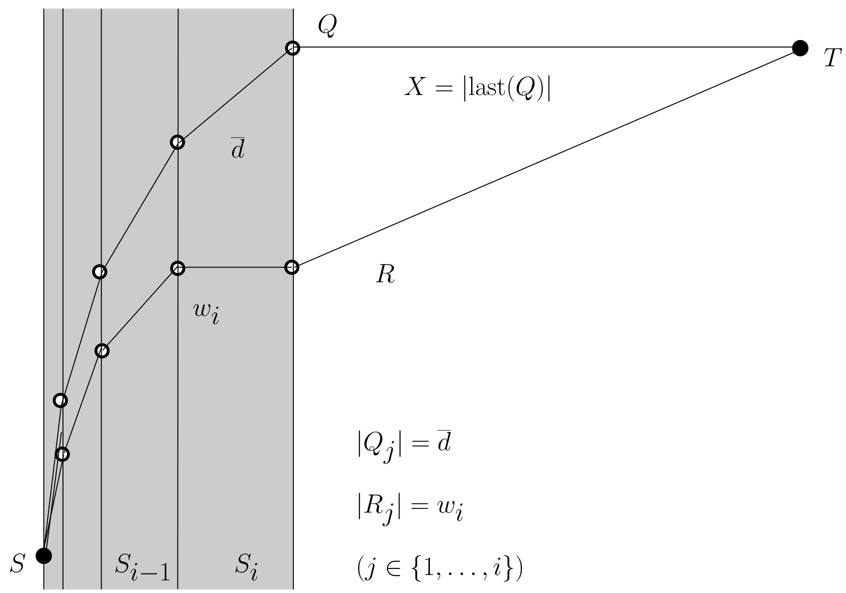

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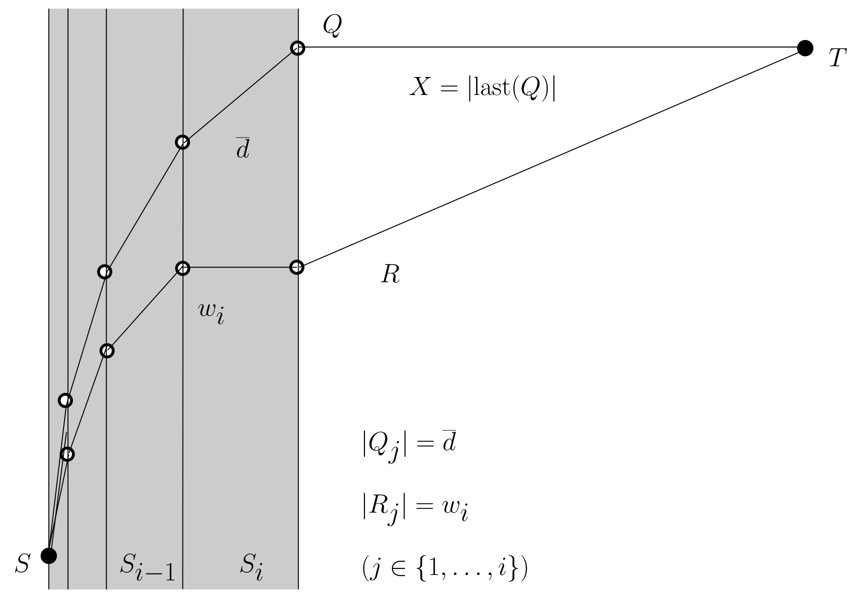

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- Find optimum P^j and d_j for $K = j$ strips in $O(j)$ time

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- Binary search: $j = \lfloor \frac{n}{2} \rfloor$
- Find optimum P^j and d_j for $K = j$ strips in $O(j)$ time
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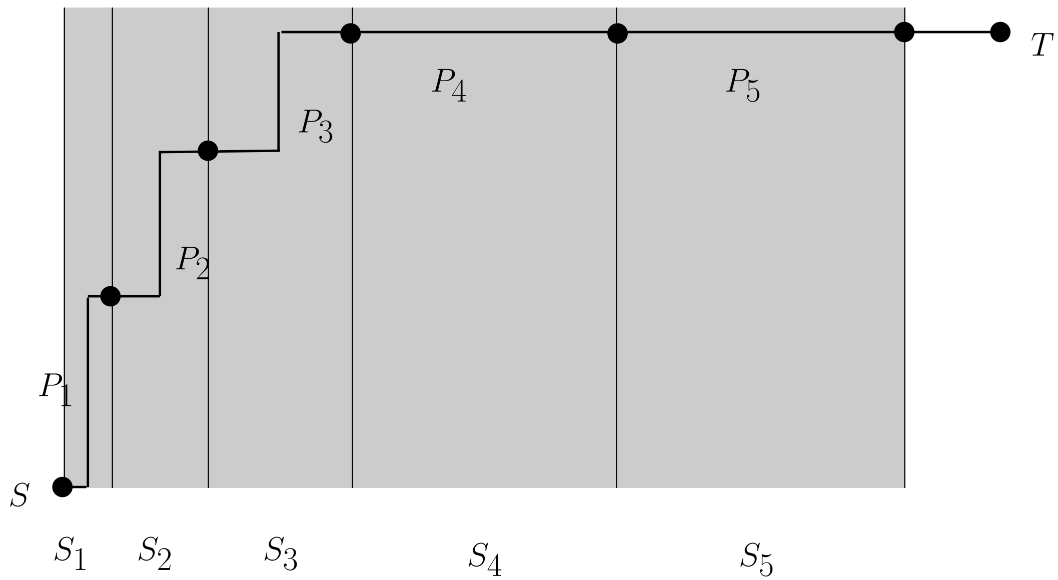
- $\Theta(n)$ space

L_1: Incrementally



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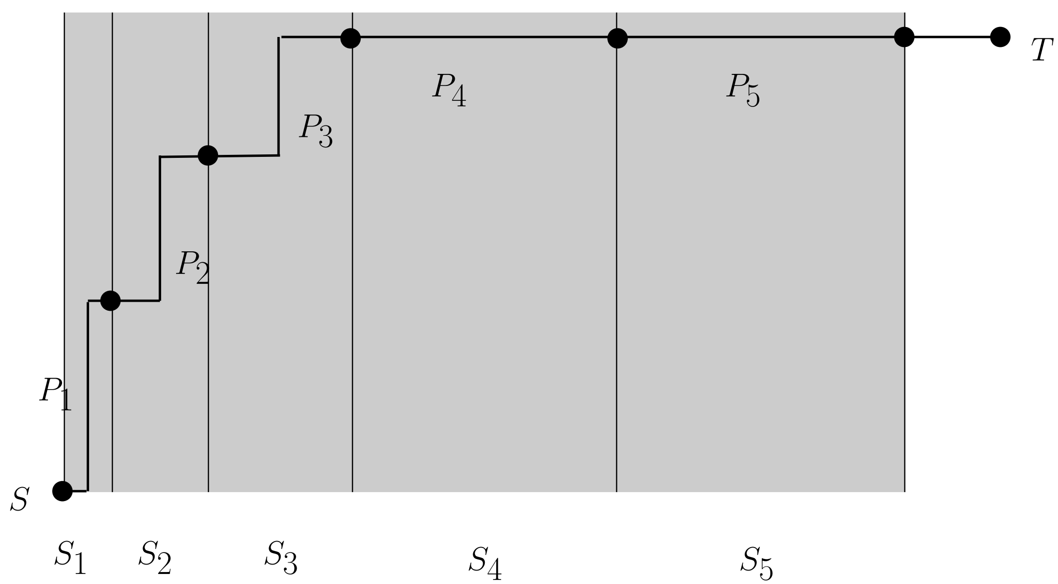

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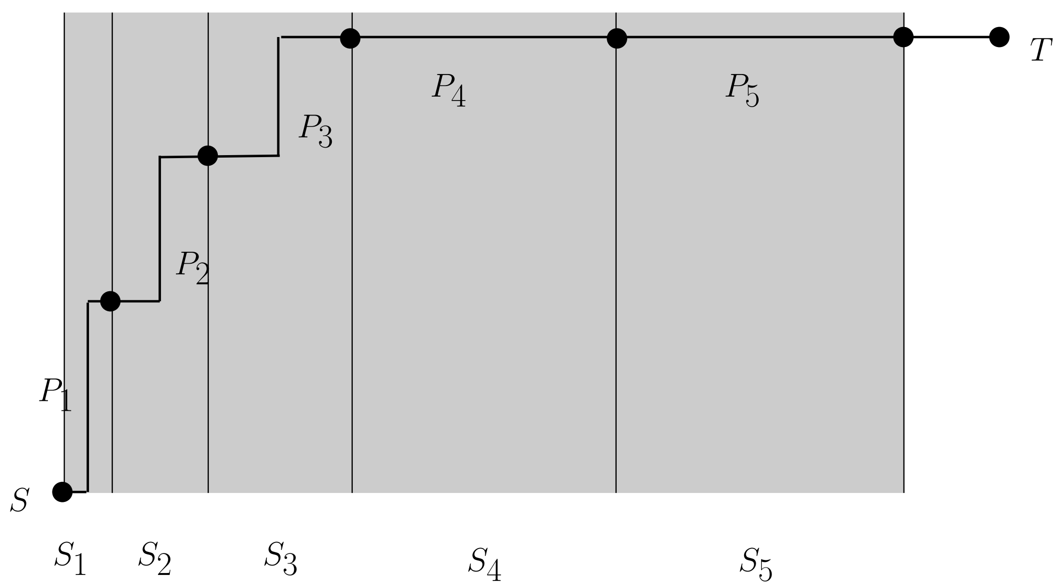

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- For $d = |P_{i-1}^{i-1}| > w_i$ we distribute $t_y + \sum_{j=1}^{i} w_j$ among i strips
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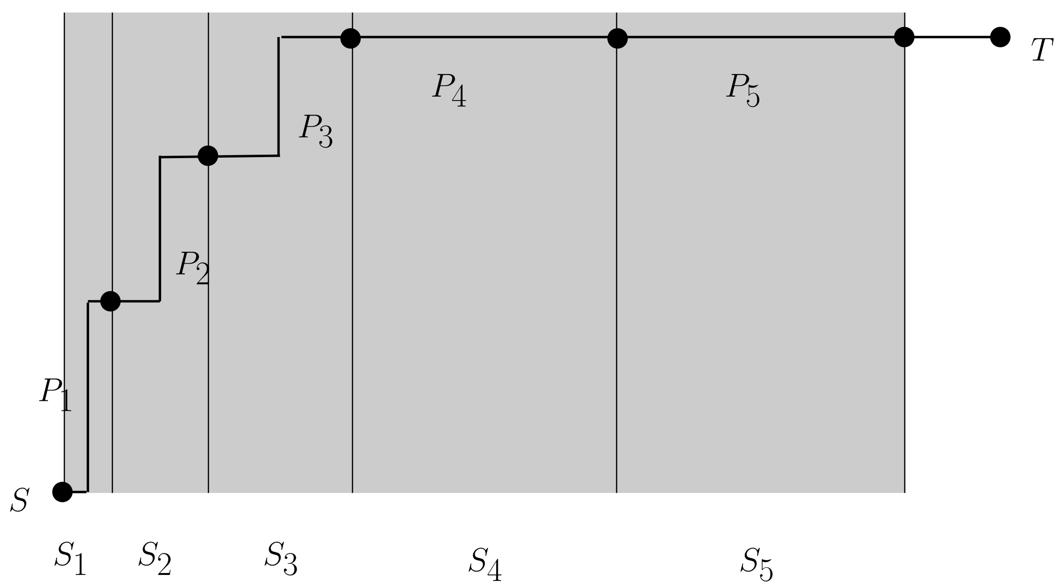

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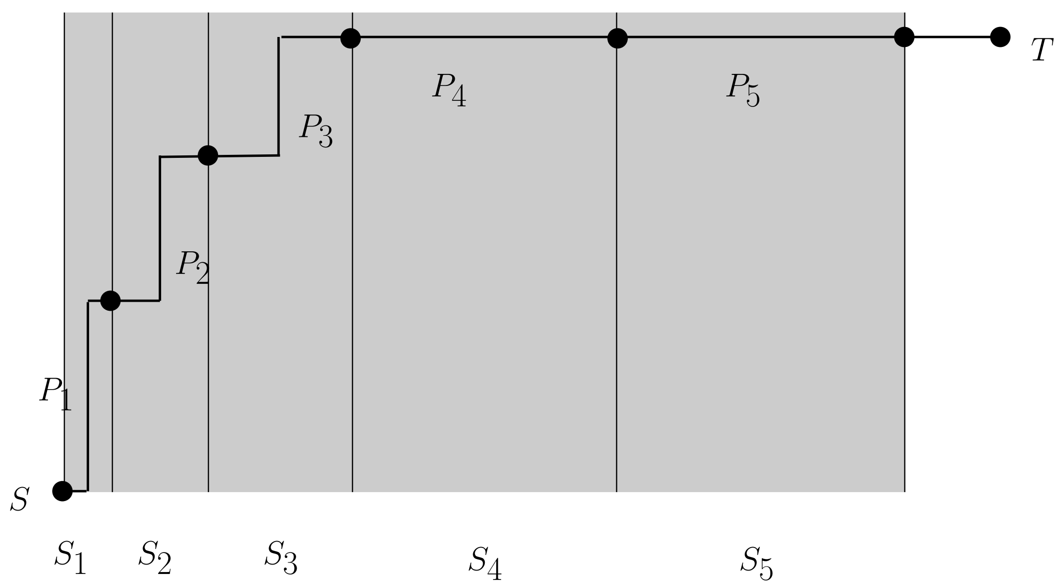
- Presorted input: Incrementally $\Theta(n)$ time and space



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