INFLUENCE OF TRANSMISSION POWER CONTROL ON THE TRANSPORT CAPACITY OF WIRELESS MULTIHOP NETWORKS

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Abstract - In this paper, we examine the general influence of transmission power control on a wireless multihop network's throughput and compare topologies resulting from different transmission power control strategies. For this purpose, we introduce concepts for measuring certain characteristics of wireless multihop network topologies.

Keywords - Wireless multihop, power control, topology control, topology characteristics, transport capacity.

I. INTRODUCTION

In recent years, wireless multihop networks have been subject to intensive research effort. The topic of topology control by adjustment of the devices' transmission powers has been of some interest, often with the motivation to increase the battery lifetime of portable network devices.

A motivation that receives less attention is that lower transmission powers promise an increased network throughput, since a higher spatial re-use theoretically outweighs longer paths: Whereas the expected path length between two stations experiences only a linear increase with decreasing transmission ranges, the spatial re-use increases quadratically in single-channel networks. In practice, things are not that simple. Therefore, the work presented here tries to give a deeper insight into the influence of the transmission power assignment on the resulting transport capacity. We consider specific scenarios resulting from given station placements and topology control strategies. This helps to reveal effects that remain undetected when analysing the asymptotic behaviour of networks under simplifying assumptions.

The rest of this paper is structured as follows: After presenting related work in section II, we describe our network model in section III. In section IV, we introduce several metrics for topology characteristics. In section V, we use these metrics to analyse different topologies. We conclude this paper by summarising our results and outlining perspectives for future work in section VI.

II. RELATED WORK

A. Capacity of Wireless Multihop Networks

Early work on the capacity of wireless multihop networks was the theoretical determination of the "magic number" of bidirectional links that stations in packet radio networks should have on average to achieve optimal throughput (e.g. [1]). This work was the first to show that transmission power control is an important issue and it gives a hint on what might be a good choice of transmission range for a given station density. However, it only deals with topologies resulting from the use of a common transmission range for all stations. Also, it uses the simplifying assumption that the traffic volume is homogeneous in all parts of the network and that at each station, the direction of a packet to the destination is equally likely to be in any direction, which certainly does not hold in many realistic scenarios.

Theoretical work [2] has shown that the throughput share for each station in a wireless multihop network declines with a growing number of stations, leading to the conclusion that ad hoc networks only scale if the traffic patterns obey a locality principle. While this work presents interesting results on the behaviour of the capacity as the number of stations approaches infinity, it does not answer the practical question how transmission ranges should be set to achieve a high capacity in a given network. The interference model and the throughput measure used in our work are adopted from [2].

In [3], the disadvantages of the IEEE 802.11 MAC layer in multihop environments are presented, showing the difficulty in interpreting results for this specific protocol. In contrast, the maximum bandwidth available to streams in special multihop topologies (chains, lattices) is analysed using a very simple model of a "perfect" MAC layer. This plain analytical approach can only be applied to very simple, regular topologies.

To our knowledge, the first approach to analyse the capacity of a specific, given topology was just recently published [4]. The authors assume that the topology and the traffic pattern (i.e., source and destination pairs) are given. The capacity is estimated as the cumulative throughput of maximal flows, subject to interference constraints represented in a so-called conflict graph. The main differences to our work are discussed in section III.

Finally, it should be noted that the work introduced so far finds spatial re-use to be the most important factor influencing the network's transport capacity, since the decrease in spatial re-use with rising transmission ranges is stronger than the decrease of route lengths. On the other hand, there is work presenting simulation results showing that the opposite

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is the case and concluding that the highest network capacity is achieved by using maximal transmission powers (e.g. [5]). This discrepancy is discussed in section V.

B. *Topology Control Strategies*

A variety of topology control algorithms has been proposed. Due to lack of space, we introduce only the strategies analysed later in this paper. For a more comprehensive presentation, refer to [6].

A straightforward topology control strategy is to assign a *common transmission range* to all stations. Its disadvantage is the high amount of control traffic necessary in decentralised operation. Other strategies are subsumed by the term *per-node* topology control.

Several distributed algorithms follow the *nearest neighbours* strategy, e.g. [6]. With this strategy, each station has the lowest possible transmission range ensuring that all stations are connected to their k nearest neighbours.

The idea of the *cone based* algorithm ([7]) is for each station to have a bidirectional link to at least one neighbour within the cone with an angle of α in every direction, given that there are any stations located there at all. The advantage is that the topologies are very likely to have a high connectivity; however, it requires a technology that directly or indirectly (e.g. via GPS) allows the devices to determine the direction in which other stations are located.

III. NETWORK MODEL

This work focusses on single-channel networks (as opposed to multiple-channel networks such as Bluetooth scatternets). Here, a single channel is used by all devices. Therefore, simultaneous transmissions have to be spatially separated to prevent interference.

Throughout this paper, S denotes the set of all stations in a network. Each station $i \in S$ has a fixed transmission range r_i. Stations $i, j \in S$ are connected if they are within each other's transmission ranges, i.e. $d_{ij} \leq \min\{r_i, r_j\}$, where d_{ij} denotes the distance between i and j. Obviously, this model requires links to be bidirectional. However, this is not essential for the work presented here; rather, link layer acknowledgements require bidirectional links, and utilising unidirectional links for routing is very complicated [8].

We measure the potential throughput of a network according to the idea of finding a good transmission schedule by running high-level simulations that model only very few, simple assumptions. Our approach is described in detail below, but first, we justify this choice.

Several metrics give a hint on the transport capacity of a wireless multihop network. As an example, the number of links per station is an indicator for the possible spatial reuse. However, calculating the possible throughput of given streams in a given network topology is extremely difficult without straightforwardly simulating network traffic: Finding an optimal transmission schedule is NP-hard [9]. Approaches for calculating lower and upper bounds of throughput are introduced in [4], but they are too complex to be practically applied to networks with a large number of stations.

On the other hand, using a simulation environment with detailed models for the different protocol layers does not produce very meaningful results here. This is due to the strong influence of shortcomings of specific MAC, routing, and transport protocols. As an example, TCP is known to show difficulties in wireless multihop scenarios.

Since both approaches have their disadvantages, we combine these methods by performing simulations on a very simple network model. Our approach is based on the idea of finding a good transmission schedule assuming an ideal, slotted MAC layer: Packet transmissions take one slot time, and within each slot, simultaneous transmissions take place only if they do not interfere with each other. Interference is modelled according to the "protocol model" [2]: A sender i interferes with the reception of another station's signal at a station j, if $d_{ij} \leq (1 + \delta) \cdot r_i$, where $\delta \geq 0$ is the so-called *guard zone*. The "physical model" also introduced in [2] may be used just as well. However, the results in this paper are limited to the protocol model, since this model is more suitable for IEEE 802.11 type networks. It may be expected that the impact of spatial re-use is smaller for the physical model.

In our model, all sources are saturated. A bottleneck-aware transport protocol is modelled by a simple flow control mechanism that allows a transmission only if the next hop does not already have more than a certain number of packets of the same stream queued. Buffering more than one packet for each stream proves to be advantageous because it allows more combinations of simultaneous transmissions, and therefore slightly improves the throughput. In the simulations presented in section V, up to four packets of each stream may be buffered at each station.

We assume that only shortest routes are used, despite the different practical disadvantages that come with this type of routing, i.e. bad signal quality over long distances and potentially short-lived links in the face of mobility. If several neighbours of a station are on the shortest path to a packet's destination, one of them is chosen as next hop randomly, with an equal probability for each. Note that this can lead to different probabilities for different paths.

Our network model does not include per link power control (which could quite easily be accomplished by a different interference model), since we are interested in results that have validity for IEEE 802.11-like networks. Therefore, the best network utilisation may be expected when defining the length of a route as the sum of all transmission distances. With this metric, shortest paths are not necessarily paths with minimum hop count, unless all stations have the same transmission range.

The reason for using shortest path routing is that routing streams over large detours is obviously a waste of resources. Of course, it may be possible to improve the throughput this way for scenarios with rather few streams (as shown in [4]). However, the economical use of resources is certainly the most important condition for a high overall throughput as the number of streams increases. Therefore, modelling shortest path routing is not a restriction.

We have experimentally verified this assumption by repeating simulations with additional low priority streams that connect the source and destination stations of the original streams via paths that are shortest while maximising the amount of free resources on each link measured in a previous simulation run. The free resources across each link were recorded as the number of slots where a transmission across this link would have been possible. For the scenarios presented in section V, no significant throughput gain could be observed, except for the scenarios with 25 streams across 500 stations, where this low number of streams is required for the comparison to the scenarios with only 50 stations.

When measuring the transport capacity of a network, fairness aspects should play an important role: The achievable overall throughput is generally maximised under highly unfair and therefore undesired conditions. For this reason, we implement a fairness strategy with a notion of max-min fairness between different streams: The fewer packets a stream has delivered at the destination, the higher is the priority of its packet transmissions. This fairness aspect is another important difference to the model used in [4].

IV. METRICS

The two main metrics that topology control algorithms trade off against each other are the network's connectivity and its transport capacity. Generally, reducing transmission powers in favour of the network capacity bears the risk of a connectivity reduction. This implies that it is reasonable to do without transmission power control and to use maximum transmission powers instead, if the network connectivity is a critical requirement. A possible scenario might be a disaster recovery operation where mobile devices are used to exchange a rather low amount of important data. In contrast, if a large number of devices forms an open ad hoc network, it can be argued that it is better to risk a lower connectivity than to have a poor network capacity in the first place.

A. Connectivity

In many scenarios, connectivity certainly is an important aspect: A device in the network should be able to reach any other device with a sufficiently high probability. Most previous work measures the probability for complete network connectedness. However, complete connectedness as a boolean property does not capture different levels of connectivity. As an example, a single device may be outside of the coverage area of all other devices, which in turn form a connected component. Although this is a disconnected network, almost all stations are able to communicate. Therefore, we refer to the *connectivity* of a network as the probability that two randomly chosen devices are able to communicate with each other:

$$C = \frac{\left|\left\{(i,j) \in S^2 : i \neq j \wedge i,\, j \text{ interconnected}\right\}\right|}{|S| \cdot (|S| - 1)}$$

B. Transport Capacity

In general, the term *capacity* refers to a maximally achievable quantity; intuitively, the *transport capacity* of a network is its highest possible aggregated end-to-end throughput.

Of course, this value strongly depends on the actual traffic pattern: It is very likely that more resources are consumed if the streams cover longer distances, resulting in a lower achievable throughput. Since a fair comparison of different topologies resulting from a given placement of stations must be based on the same traffic patterns, specific patterns have to be chosen which are used to evaluate different topologies.

Another implication of this dependency on the source-destination distances is that simply measuring the overall throughput in bits per second is quite meaningless. Rather, it is useful to measure the total throughput weighted by the end-to-end distance of each stream:

$$T = \sum_{i \in S} \sum_{j \in S} t_{ij} \cdot d_{ij},$$

where t_{ij} denotes the throughput from i to j (in data units divided by time units).

Taking the distance into account when calculating the throughput was introduced in [2] (presented there as *bit-metres per second*). For the sake of brevity, the term *throughput* is used to denote this "distance-throughput" throughout the rest of this paper.

C. Route overhead

To measure the unnecessary consumption of resources that is caused either by even shortest routes taking considerable detours or by unnecessarily high transmission ranges, we introduce an overhead metric as

$$O = \frac{\displaystyle\sum_{i \in S} \sum_{j \in S \setminus \{i\}} \left[\frac{w_{ij}}{|R_{ij}| \cdot d_{ij}} \sum_{k \in S \setminus \{j\}} |R_{ij} \cap R_k| \cdot r_k \right]}{\displaystyle\sum_{i \in S} \sum_{j \in S \setminus \{i\}} w_{ij}} - 1,$$

where R_{ij} denotes the set of all shortest routes between stations i and j, and R_k denotes the set of all shortest routes including a station k. All shortest paths between two stations are assumed to be equally likely, while the weight function w gives each source-destination pair a different weight. Generally, $w_{ii} = 0$ for all $i \in S$. Also, we assume $w_{ij} = 1$ for all $i \neq j$ throughout the rest of this paper, which means that all station pairs are equally likely to communicate.

The main fraction in this formula is the weighted average over all source-destination pairs of the quotient between the total distance of the transmissions required to transport a packet, averaged over all possible shortest routes, and the source-destination distance. By subtracting 1, we obtain the actual overhead, i.e. the distance covered unnecessarily. Note that this formula is not restricted to shortest paths according to a certain path cost metric. Actually, R_{ij} and R_k can be defined arbitrarily. This metric is calculated in $O(n^3)$ by an extension of Floyd's all pairs shortest paths algorithm.

Other literature often uses the "stretch factor" to measure the unnecessary consumption of resources. This metric captures a worst case, but just as argued previously on the connectivity issue (section IV-A), it is more useful in this context to consider the overhead that can be expected rather than an arbitrarily unlikely worst case. Also, the concept of the stretch between stations fits better to networks using additional per-link power control.

V. RESULTS

In this section, we apply the metrics introduced in section IV to examine the impact of topology control strategies and their parameters. The scenarios consist of a number of stations that are uniformly distributed on a square area with a side length of $\sqrt{0.5}$km, so the maximal distance between two stations is 1km. We use a guard zone of $\delta = 1$. The results we present are not qualitatively influenced by the choice of this parameter within reasonable bounds. The traffic consists of saturated streams, where each source-destination pair is equally likely. All figures show the 0.95 confidence intervals of the respective mean values.

A. Varying common transmission ranges

Figure 1 shows the total throughput of 25 streams in scenarios with 50 stations compared to that in scenarios with 500 stations, depending on the common transmission range used. The figure shows that in the scenarios with 50 stations, it is not possible to achieve a noteworthy capacity gain by increasing the spatial re-use, whereas it very well is possible in the scenario with 500 stations (the throughput for a common range of 75m is almost twice the throughput with a common range of 300m). The reason for this discrepancy becomes clear when taking a closer look at how the network capacity is influenced by the common transmission range used by the stations (see figure 1(c)).

For small transmission ranges resulting in rather low connectivity values, connectivity is the main factor influencing the capacity of the network, since a higher connectivity means that a higher number of streams can be established in the first place. Therefore, the capacity increases with rising transmission ranges.

If the network connectivity is high enough for most of the streams to be established, the decline in spatial re-use (which is asymptotically quadratic) outweighs the reduced path length (which is asymptotically linear) for higher transmission ranges, so the spatial re-use is the main factor for the capacity. Therefore, the capacity now decreases with rising transmission ranges.

However, if the transmission ranges reach such a high level in relation to the size of the area that there is hardly any room left for spatial re-use, the route lengths continue to decrease. Therefore, the route lengths are the main influencing factors here, and the capacity increases again with rising transmission ranges.

In figure 1(c), the three "zones" where a different factor is dominant for the network capacity are labelled A, B, and C. Clearly, the transition between A and B, i.e. the lower bound on the transmission ranges for a network with a high connectivity, mainly depends on the station density. In contrast, the transition between B and C, i.e. the upper bound on the transmission ranges such that a single transmission does not block the majority of all other stations, depends on the size (and shape) of the area itself. Consequently, zone B will completely disappear if the number of stations is low enough, because a sufficient network connectivity is reached only when transmission ranges are so high that there is no possibility of spatial re-use.

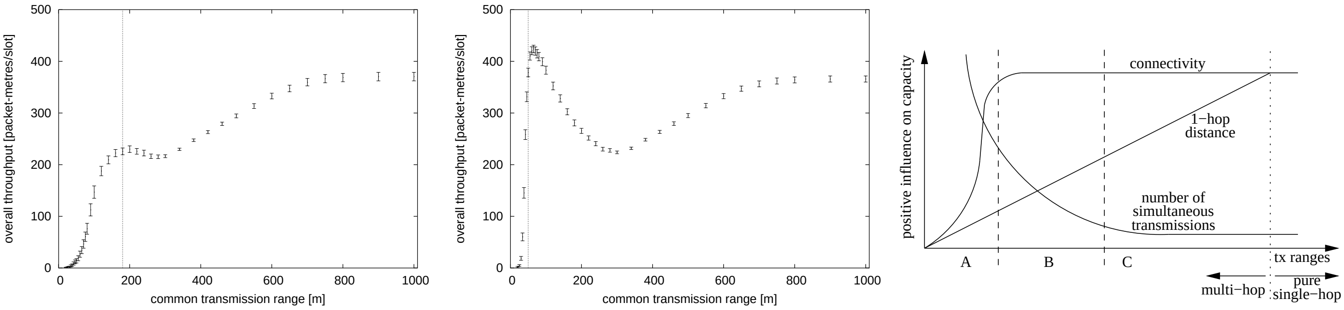
This explains the fundamental difference between the throughput plots in figures 1(a) and 1(b), as confirmed by the dashed lines, which show where a connectivity of 0.95 is reached on average. This is most probably also the reason why simulations presented in other work with a rather low number of stations suggest that the capacity rises monotonically with increasing transmission ranges (e.g. [5]). In ns-2 scenarios with a higher number of stations (e.g. 500 as presented in [6]), the advantage of higher spatial re-use with lower transmission ranges can very well be observed.

B. Influence of the topology control strategy

Now, we apply three different topology control strategies (cf. section II-B): The common range strategy, the nearest neighbours strategy, and the cone based strategy. The strategies are implemented with global knowledge such that the transmission ranges assigned to the stations are minimal while fulfilling the strategies' requirements.

Figure 2 compares the throughput of 250 streams in topologies created according to different strategies for networks with 1000 stations. We observe that the behaviour of the throughput for the per-node strategies is similar to that of the common range strategy described in the previous section. In particular, the maximum achievable throughput is approximately the same for all three strategies.

Although the cone based strategy is actually intended for values of $\alpha \leq 2/3\pi$ (at least 3 cones in 2π) which guarantees connected topologies, we have also considered topologies with larger cones. Figure 2(c) reveals a strange effect: Around $\alpha = \pi$, the capacity of the topologies is slightly decreased. This is caused by stations with extremely high transmission ranges near the border of the area and is a drawback of the cone based strategy itself that might only

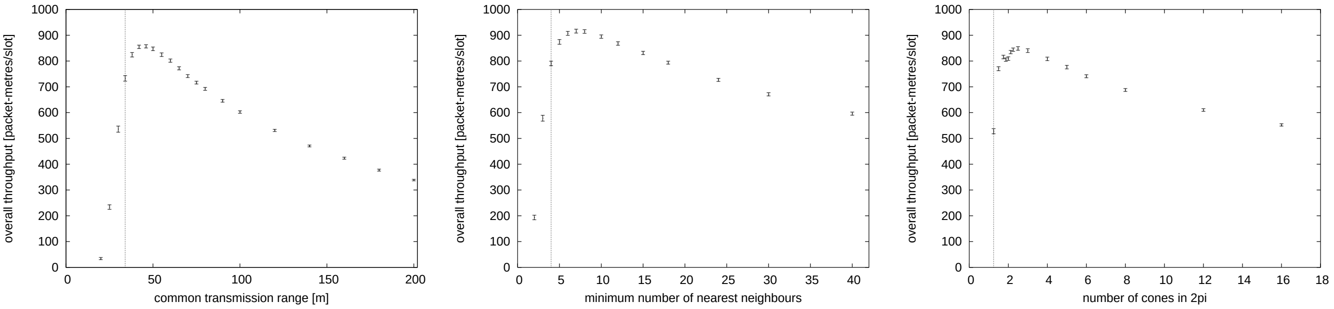


(a) 50 stations

(b) 500 stations

(c) Factors influencing the network capacity (sketch)

Fig. 1. Throughput depending on the number of stations (common ranges, 25 streams)



(a) Common ranges

(b) Nearest neighbours

(c) Cone based

Fig. 2. Throughput depending on the topology control strategy (1000 stations, 250 streams)

be eliminated by introducing more complex rules. It is worth noting that while this drawback is obvious around $\alpha = \pi$, it can also be observed for other values of α, as we will see in the discussion of figure 4. Also, it is important to note that the shortest path metric used here absorbs the negative influence quite well, since stations with unnecessarily high transmission ranges are avoided. (This has been verified by comparison to simulations using minimum hop count paths, where the dent in the throughput plot becomes drastic.)

Figures 2(a) and 2(b) show that for topologies with a just barely sufficient connectivity, increasing the transmission ranges still has a positive effect on the network capacity before the negative impact of the reduced spatial re-use as described in section V-A finally prevails. This is due to the high overhead associated with topologies that are only very loosely connected, as confirmed by the topologies' overhead values (cf. section IV-C). Figure 3 shows these values for the common range strategy: While we observe extremely high overhead values for networks with a low connectivity, the average overhead is approx. 0.62 for a common range of 34m, as opposed to only 0.32 for 50m.

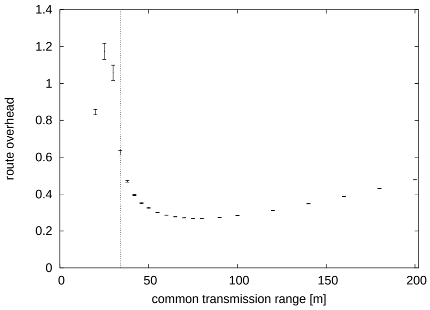


Fig. 3. Route overhead

Figure 4 shows the ratio between unidirectional links and bidirectional links for the cone based strategy. The plot for the nearest neighbours strategy has been omitted, since this ratio is close to 0.2 for all parameters we have simulated. It can be seen that the cone based strategy produces much more unidirectional links than the nearest neighbours strategy, in other words, the transmission ranges are much more inhomogeneous. Obviously, the cone based strategy results in very

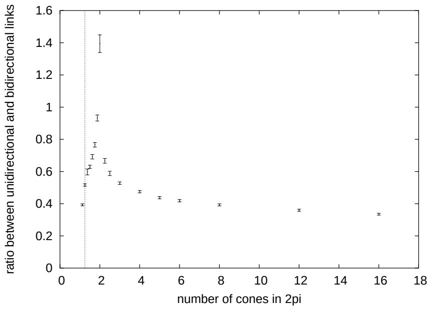


Fig. 4. Ratio of unidirectional and bidirectional links

high transmission ranges for some nodes for all parameter choices. This disadvantage may become less drastic when placing an upper bound on the transmission range, as it is the case in reality. Nevertheless it is worth noting that this strategy does have some difficulties.

VI. CONCLUSION & FUTURE WORK

In this paper, we have introduced some practical metrics for wireless multihop network topology characteristics that are important for understanding the relation between transmission power control and the resulting transport capacity.

We have shown that controlling devices' transmission powers to increase a wireless multihop network's spatial re-use has a positive effect on its transport capacity, given that the network is large enough. For all topology control strategies considered, raising the transmission ranges beyond a certain point reduces the overall throughput.

We have seen that with favourable parameter choices, all three strategies result in topologies which are comparable in terms of the achievable throughput. It is interesting to note that the utilisation of directional information as implemented in the cone based strategy results in topologies with much more inhomogeneous transmission range assignments than with the simple nearest neighbours strategy. While the idea of using directional information is intuitively reasonable, more effort must be put into the design of such a strategy. This leads to the conclusion that the choice for a certain strategy can be based on practical issues.

Furthermore, transmission powers higher than necessary for a high connectivity are often advantageous for the network capacity due to the lower route overhead. This result is interesting since the primary goal of transmission power control in other work is often to create topologies that are just barely connected in order to save scarce battery power. However, we believe that the route overhead ought to be taken into account from an energy conservation point of view just as well.

Future work will cover some interesting aspects that were not yet considered: The influence of different fairness notions is probably not too severe on the aggregated throughput. However, a closer look remains to be taken on the bandwidth share single streams can expect. In this context, the fairness modelled will play an important role.

While our simulation model shows the potential of a given topology, the impact of the hidden node problem remains unclear. Even with a common transmission range, the virtual carrier sense mechanism (RTS/CTS exchange) is not able to completely overcome this problem. From the results presented here and in other work, it is not clear that this problem necessarily aggravates with per-node transmission power control, as it is often suspected.

Finally, we have only considered uniform station distributions in this work, for the placement of stations as well as the choice of source-destination pairs. The impact of other spatial distributions of stations and streams is an open issue.

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